THE STRONG TRACKING AUGMENTED KALMAN FILTER FOR SATELLITE ATTITUDE DETERMINATION

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ABSTRACT

For parameter estimations, we have developed a extended kalman filter(EKF) and unscented kalman filter(UKF) for linear as well as non-linear spacecraft systems. We have described the differences of two approaches mathematical equation for modeling of the system value of mean square error, error covariance and For state estimation of satellite, two different type of filters has been described i.e, extended kalman standard deviation are proposed to be used for determining the accuracy of implemented method.

KEYWORDS: Filtering Algorithm, Non-Linear System, Strong Tracking, State Estimation, Unscented kalman Filter.

I. INTRODUCTION

Satellite pictures are used for many applications such as reconnaissance and geographic information systems. So, requirement of operation and design of satellite systems have become more important and greater system reliability during operation is required. All the practical systems posseses some degree of non-linearityDepending on the type of process and the operating region of the process, some processes can be approximated with a linear model and KF can be used for state estimation. For the non-linear systems, under the assumptions of Gaussian noise, the extended kalman filter (EKF) is frequently used for estimating the non-measurable state variables through the processing of input and output sequence. The EKF is typically based on linearization of the system dynamics using the first order taylorexpansion. There are many uncertainties to deal with process control; model uncertainties, measurement uncertainties and uncertainties in terms of different noise sources acting on the system. The linear approximation of the system at a given time instant may introduce errors in the state which may lead the state to diverge over a time. In other words the linear approximation may not be appropriate for some systems. In order to overcome the drawbacks of the EKF ,other non-linear state estimators have been developed such as unscented kalman filter(UKF) or the higher order EKFs.The state distribution in UKF is approximated by Gaussian random variables ,which is represented by using a minimal set of suitably chosen weighted sample points. these sigma points are propagated through the true non-linear system, thus generating the posterior sigmapoint set ,and the posterior statictics are calculated. the sample points progressively converge to the true mean and the covariance of the guassian random variables.

Henry D. Travis performed the attitude determination task using a linear Kalman Filter alongwith star tracker measurements on satellites. The filter used a linear, constant coefficient statematrix with the optimal control law to provide negative feedback control. The control law usedinformation developed through the equations of motions of the spacecraft in the orbit. Thesystem assumed the position and motion of the satellite as linear. When the Kalman Filter algorithm was applied to the satellite systems, the algorithm described good results in terms oferror covariance, mean square error and standard deviation The Extended Kalman Filter provided an improvisation in calculating the estimation of the state of a non-linear system over a Linear Kalman Filter For improving and implementing the method, the various study steps have been described below. Firstly, Analysis of various orbital parameters that are intended for defining a satellite in any orbit. Secondly, Analysis of the three axes symmetry of the satellite body that define the attitude of the satellite in the orbit. Thirdly, Development of mathematical equations for modeling of the system and applying it for parameter estimation. The value of mean square error, error covariance and standard deviation are proposed to be used for determining the accuracy of the implemented method. Lastly, Simulation of the above.

II.KALMAN FILTER MECHANICS

In estimating the state of discrete-time controlled process governed by the linear stochasticdifference equation,

$x_k = A\, x_{k-1} + B\, u_{k-1} + w_{k-1}$…………….…………2.1

The Kalman filter deals with the general problem with a measurement that is,

$Z_k = H\, x_k + v_k$………………………………2.2

The process and measurement noise are represented by the random variables w_k and $v_k$respectively and are independent of each other (assumed), with normal probability distributions

$p(w) \sim N(0, Q)$………………..………………2.3
$p(v) \sim N(0, R)$……………………………………2.4

In the difference equation A is an n×n matrix which relates the previous time state at step k-1to the current state at step k whereas B is an n×l which relates the state x to the optional controlinput. And in the measurement equation H is an m×n matrix which relates the state to the measurement z_k.

The Kalman filter uses a form of feedback control in estimating a process. The filter acquires feedback as (noisy) measurements by first estimating the state of the process at some time. As a result, the Kalman filter equations for the process diverge into two groups: the time update equations and the measurement update equation.

The time update equations accountfor driving the present state and error covariance estimates ahead in time for obtaining the pastestimates for the next time step. The measurement update equations account for the feedback. For assimilating a fresh, new measurement into the past estimate for obtaining a superiorsubsequent estimate. One can also thought the time update equations as predictor equations,and the measurement update equations as corrector equations.The specific equations for the time and measurement updates are presented.,

Kalman filter time update equations:

$\mathbf{\hat{x}_k^- = A\, \hat{x}_{k-1} + B\, u_{k-1}}$…………………………2.5

$\mathbf{P_{K}- = A\, P_{K-1} A^{T} + Q}$……………………………....2.6

Kalman filter measurements update equations:

$\mathbf{KK = P_{K}- H^{T} (HP_{K}-H^{T} + R)^{-1}}$………………………..2.7

$\mathbf{X\wedge_{K} = X\wedge_{K}- + KK (z_{K}- H\, X\wedge_{K}-)}$…………………………

$\mathbf{P_{K} = (1- KKH)\ P_{K}-}$……...………………………2.9

The computation of the Kalman gain KK. is the initial step during the measurement update compute Subsequent to the above step is to measure the process for obtaining the measurement z_K , and then generating an a posteriori state estimate by assimilating the measurement. The process is concluded by obtaining an a posteriori error covariance estimate. This process is reiterated after every time and measurement update pair with the previous "aposteriori" estimates for predicting the new "a priori" estimates. As a result of this recursive nature of the Kalman Filter it makes practical implementations a lot more feasible and is one ofthe very pleasing attributes of the filter. The KF in its earliest formulation can be seen as asequential least-squares approach for estimating longitudinal factor scores when no prior information is available. In other instances, the KF is often used in conjunction with amaximum likelihood procedure termed prediction error decomposition to estimate parametersfor dynamic and time series models. Central to the prediction algorithm of the KF is a state space model that specifies the dynamic and measurement relations among latent states and manifest observations.

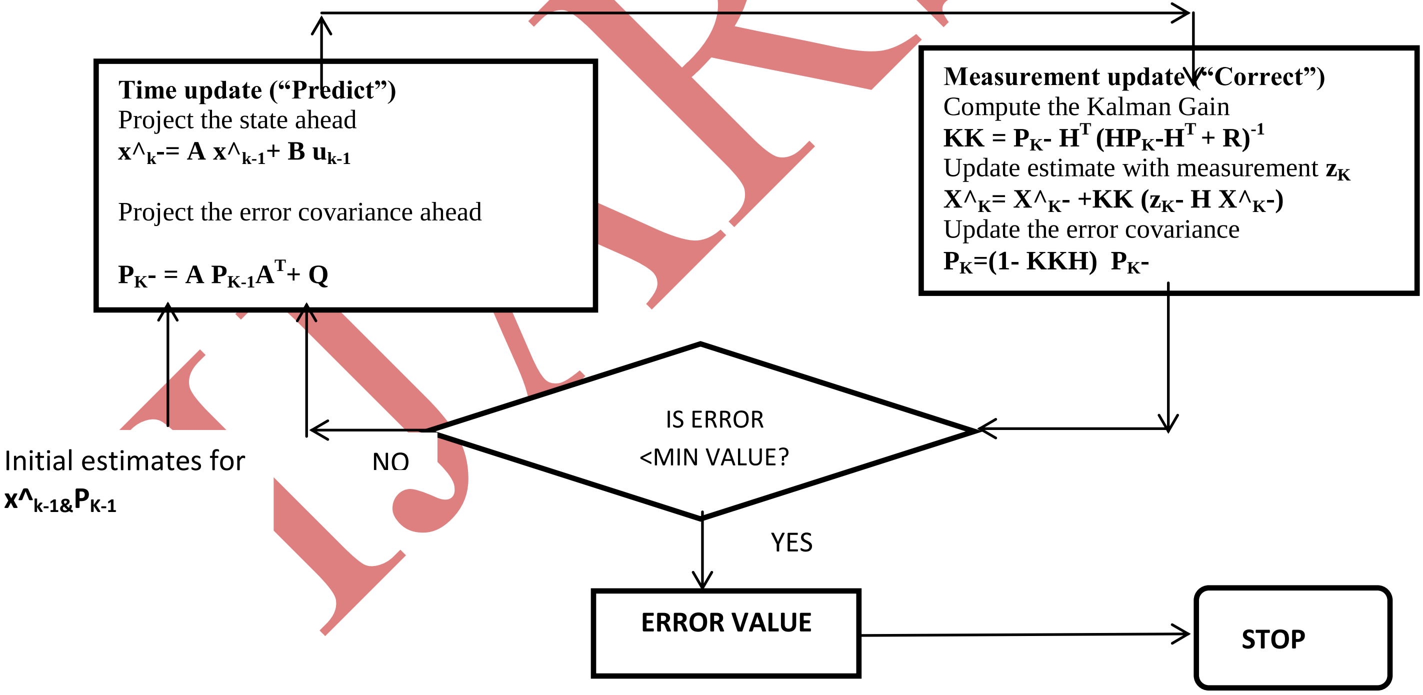

Figure 2.2- A complete picture of the operation of the Kalman filter

III. UNSCENTED KALMAN FILTER ALGORITHM

The UKF was founded on the intuition that it is easier to approximate a probability distribution that it is to approximate an arbitrary nonlinear function or transformation. The sigma points are chosen so that their mean and covariance to be exactly x^a_{k-1} and p_{k-1}. Each sigma point is thenpropagated through the nonlinearity yielding in the end a cloud of transformed points. The new estimated mean and covariance are then computed based on their statistics. This process is called unscented transformation. The unscented transformation is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation.

Consider the following nonlinear system, described by the difference equation and the observation model with additive noise:

$x_k= f\, x_{k-1}+ w_{k-1}$……………..……………3.1

$z_k = H\, x_k + v_k$………………………………3.2

The initial state x_0 is a random vector with known mean,

$\mu_0= E\, [x_0]$ …………………………………..3.3

And covariance

$P_0=E[(x_0-\mu_0) + (x_0 -\mu_0)^T]$………………………..3.4

In the case of non-additive process and measurement noise, the unscented transformation scheme is applied to the augmented state:

$x_k^{aug} = [x_k^T w_{k-1}^T v_k^T]$……………………………..3.5

3.1.1 Set Selection of Sigma Points:

Let x_{k-1}be a set of 2n + 1 sigma points (where n is the dimension of the state space) and their associated weights:

$X_{k-1} = \{(x^j_{k-1} , W^j) / j= 0……..2n\}$ …………………..3.6

$x^0_{k-1} = x^a_{k-1}$ ……………………………………..3.7

$-1 < W^0>1$

$X^i_{k-1} = x^a_{k-1} + [\{n/(1-W^0)\}^{1/2} P_{k-1}]_i$, for all i = 1 to n………………..3.8

$X^{i+n}_{k-1} = x^a_{k-1} - [\{n/(1-W^0)\}^{1/2} P_{k-1}]_i$, for all i = 1 to n…………………3.9

$W^j= (1- W^0) / (2n)$, for all j = 1 to 2n ………..……………3.10

where the weights must obey the condition:

$\Sigma^{2n}_{j=0}W^j= 1$, …………………..…………3.11

And $[\{n/(1-W^0)\}^{1/2} P_{k-1}]_i$,is the row or column of the matrix. W^0controls the position of sigma points. $W^0\geq 0$ points tend to move further from the origin, $W^0\leq 0$ points tend to be closer to the origin.

IV. STRONG TRACKING AUGMENTED UNSCENTED KALMAN FILTER (ST-AUKF)

The traditional UKF algorithm requires the terms that the system noise must be the white Gaussian noise which do not participate in most of the nonlinear system. These requests limit the usage of the traditional UKF algorithm. Merwe proposed, augmented UKF (AUKF) algorithm to solve the problem of filtering the nonlinear system with the non-additive white Gaussian noise[26].The AUKF algorithm uses the process noise and measurement noise as the system state to sampling the Sigma points which resolve the problem above. AUKF algorithm uses the minimum covariance estimation principle as the basic theory. It requires that, system model and the noise statistics should be known precise, otherwise the state estimation will be inaccurate and filter outputs will be divergent. The accuracy of MEMS sensors will decrease fast while the platform moves speediness, which result in the filter losses the tracking of the states. Zhou Donghua proposed the Strong Tracking Filter (STF) [27]. STF constrains the innovation outputs of filter to satisfy the orthogonality principle by the adaptive fading factor to make the filter has the robustness performance of uncertain of the system model.

4.1 Strong Tracking Filter Theory

Considering the discrete nonlinear system as follows.

$\mathbf{X}_{K+1} = \boldsymbol{\varphi}_{K+1/K}\,\mathbf{X}_K + \mathbf{W}_K$……………………………………………….4.1

$\mathbf{Z}_{K+1} = \mathbf{H}_{K+1}\,\mathbf{X}_{K+1} + \mathbf{V}_{K+1}$………………………………………….4.2

Where $\mathbf{X}_K$ and $\mathbf{Z}_K$, denote the state vector and measurement vector, respectively; U_K, denotes the input vector; $\Phi_{K+1/K}$ and H_{K+1} denote the states transfer matrix and the measurement matrix, respectively; $\mathbf{W}_K$ and $\mathbf{V}_K$ are independent white Gaussian noise with the statistics as follows:

$E(\mathbf{W}_K) = 0$, $Cov(\mathbf{W}_K, \mathbf{Wj}^T) = \mathbf{Q}_K\,\boldsymbol{\rho}_{k,j}$………………………..4.3

$E(\mathbf{V}_K) = 0$, $Cov(\mathbf{V}_K, \mathbf{V}_j^T) = \mathbf{R}_K\,\boldsymbol{\rho}_{k,j}$…………..……………..4.4

$Cov(\mathbf{W}_K, \mathbf{V}_j^T) = 0$ …………..……………………………………….4.5

Where $\mathbf{Q}_K$ and $\mathbf{R}_K$ are positive definite symmetry matrix; the initial state $\mathbf{X}_0$ is independent with $\mathbf{W}_K$ and $\mathbf{V}_K$ with the Gaussian distribution.

The STF is similar with the traditional Kalman filter, merely using the adaptive fading factor to modify the prediction of the error covariance to maintain the innovation to satisfy the orthogonality principle. The process of prediction of the error covariance with the adaptive fading factor is:

$$\mathbf{P}_{k+1/k} = \boldsymbol{\lambda}_{k+1}\boldsymbol{\varphi}_{k+1,k}\mathbf{P}_k\,\boldsymbol{\varphi}_{k+1,k}^{T} + \mathbf{Q}_K \quad \text{……………………………….4.6}$$

Where $\boldsymbol{\lambda}_{k+1} \geq 1$ denotes the adaptive fading factor. Assume that the theoretical output innovation is $\boldsymbol{\epsilon}_{k+1} = \mathbf{Z}_{K+1} + \check{\mathbf{Z}}_{K+1,k}$, then the adaptive fading factor $\boldsymbol{\lambda}_{K+1}$ ' s calculation follows the algorithm:

$$\boldsymbol{\lambda}_{K+1} = \begin{cases} \boldsymbol{\lambda}_0, & \boldsymbol{\lambda}_0 \geq 1 \\ 1, & \boldsymbol{\lambda}_0 < 1 \end{cases} \quad \text{……………………………………………….4.7}$$

$$\boldsymbol{\lambda}_0 = \mathbf{tr}\,[\mathbf{N}_{K+1}] \,/\, \mathbf{tr}\,[\mathbf{M}_{K+1}] \quad \text{…………………………………….4.8}$$

$$\mathbf{N}_{K+1} = \mathbf{V}_{K+1} - \mathbf{H}_{K+1}\,\mathbf{Q}_K\,\mathbf{H}_{K+1}^{T} - \mathbf{R}_{K+1} \quad \text{…………………………..4.9}$$

$$\mathbf{M_{K+1} = H_{K+1}\, \varphi_{K+1/K}\, P_K\, \varphi_{K+1}^T\, H_{K+1}^T} \ldots\ldots\ldots\ldots 4.10$$

Where arithmetic operators tr (.) represents the trace of a matrix. $P^{(l)}_{K+1/K}$ denotes the prediction of the error covariance without the adaptive fading factor. Obviously:

$$\mathbf{P^{(l)}_{K+1/K} = \varphi_{K+1,K}\, P_K\, \varphi_{K+1,K}^T + Q_k} \ldots\ldots\ldots\ldots 4.11$$

$\mathbf{V_{K+1}}$denotes the covariance matrix of the real output innovation at the k +1th sampling instant, which estimated by the following formulation:

$$\mathbf{V_{K+1}} = \begin{cases} \epsilon_1 \epsilon_1^T, & K=0 \\ \rho V_K + \epsilon_{K+1} \epsilon_{K+1}^T / (1+\rho), & K\geq 0 \end{cases} \ldots\ldots\ldots\ldots \mathbf{4.12}$$

$0 < \rho \leq 1$ denotes the adaptive factor, usually assume $\rho = 0.95$.

The theory of the STF shows that the essence of the STF is to constrain the output innovation to satisfy the orthogonally principle by the adaptive fading factor.

V. NUMERICAL SIMULATION

In this section, we describe numerical simulation used to verify the performance of the proposed attitude estimation algorithm based on strong tracking augmented unscented kalman filter (ST-AUKF). AUKF algorithm uses the minimum covariance estimation principle as the basic theory. It requires that the system model and the noise statistics should be known precise, otherwise the state estimation will be inaccurate and filter outputs will be divergent. STF constrains the innovation outputs of the filter to satisfy the orthogonality principle by the adaptive fading factor to make the filter has the robustness performance of uncertain of the system model. The values of error in terms of mean square error, error covariance and standard deviation have been given in Table.1.

First fig.1 shows the orbital parameters of a microsatellite. For simulation of attitude determination using Strong Tracking Augmented Unscented Kalman Filter the orbital parameters of microsatellite have been processed through MATLAB 7.6.0.The orbit of the satellite is a near circular sun-synchronous polar orbit with an eccentricity of0.001075. The orbit is retrograde, meaning that the satellite moves from east to west, with aninclination of 98.540. The semi-major axis of the orbit is 7071 km and relates to an orbitalheight of approximately 792 km. The satellite has a mean motion of 14.322 revolutions per dayin this orbit with an orbit period of 6012 seconds.

Second fig.2 shows the attitude error of a satellite attitude measurements. This is an error which signifies the error in the pointing direction of the satellite body axis when the satellite's angular velocity changes.Figure.2 shows the pointing error of the roll axis of the satellite body. Pointing error of roll axis signifies the movement of satellite's antenna north and south. The dip and rise in the portion of the time from 24s to 30s signifies a large value of error as a result of change inangular velocity of the satellite.

Third fig.3 shows the pointing error of the yaw axis of the satellite body. Pointing error ofnick/yaw axis signifies the rotation of satellite's antenna. The dip and rise in the portion of thetime from 24s to 30s signifies a large value of error as a result of change in angular velocity ofthe satellite.

Fourth fig.4 shows the pointing error of the pitch axes of the satellite body. Pointing error ofpitch axis signifies the movement of satellite's antenna east and west. Here also the dip and risein the portion of the time from 24s to 30s signifies a large value of error as a result of change inangular velocity of the satellite.

Fifth fig.5 shows the velocity error of the satellite body. This is an error which signifies the change in the velocity of satellite's body axis when satellite's angular velocity changes.it shows an error which signifies the change in the velocity of satellite's body axis when satellite's angular velocity changes.

Sixth fig.6 shows the error in the rate of change of pitch axis with respect to time. Velocity error of pitch axis signifies the change in the velocity of satellite's antenna in east-westdirection with respect to change in the angular velocity of the satellite.

Seventh fig.7 shows the error in the rate of change of yaw axis with respect to time. Velocity errorof yaw axis signifies the change in the velocity of satellite's body rotation with respect tochange in the angular velocity of the satellite.

Eight fig.8 shows the position error of the satellite body. This is an error which signifies the change in the position of the satellite body axis when the satellite changes its position.it is the error in the change of direction of roll axis. Position error of roll axis signifies the displacement of satellite's antenna in north-south direction when the satellite is subjected to a change in its angular velocity.

Ninth fig.9 shows the error in the change of direction of roll axis.Position error of roll axissignifies the displacement of satellite's antenna in north-south direction when the satellite issubjected to a change in its angular velocity.

Tenth fig.10 shows the error in the change of direction of yaw axis. Position error of yaw axissignifies the rotation of satellite's antenna when the satellite is subjected to a change in itsangular velocity.

Based on the above analysis of the pointing error of the satellite body obtained through thecomputer simulation of the implemented method the Table No.1 describes the various errorparameters that have been obtained.

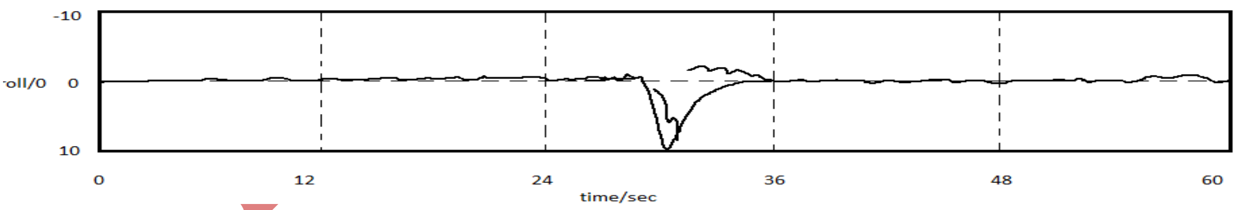

Fig2-Roll Error

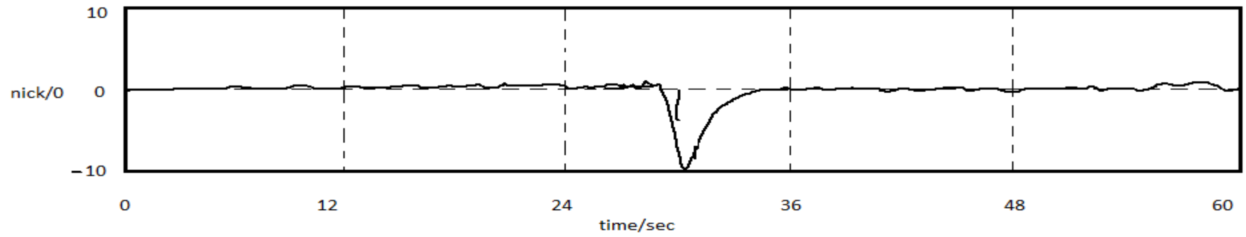

Fig3-Nick/Yaw Error

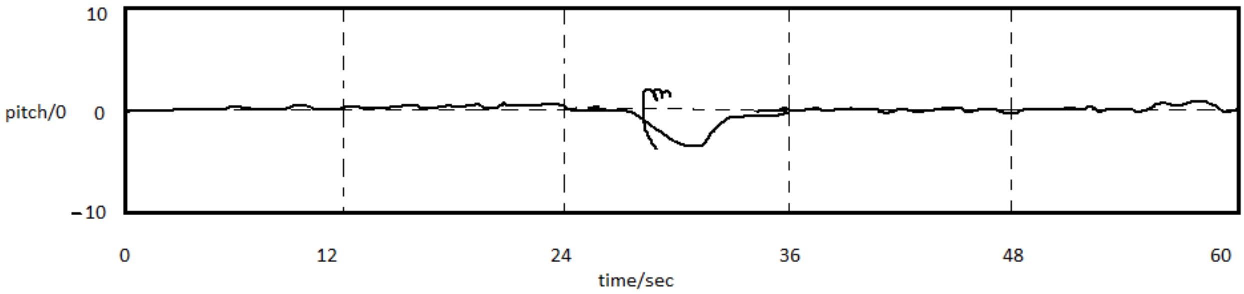

Fig4- Pitch Error

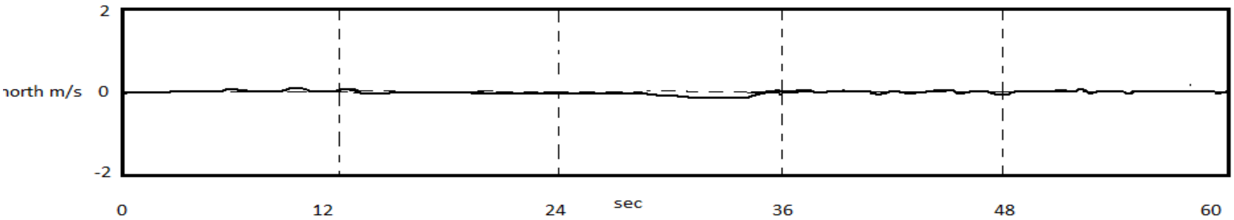

Fig5- North Velocity Error

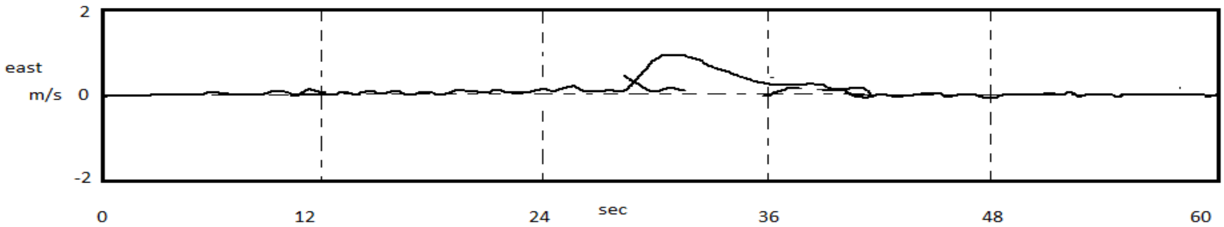

Fig6- East Velocity Error

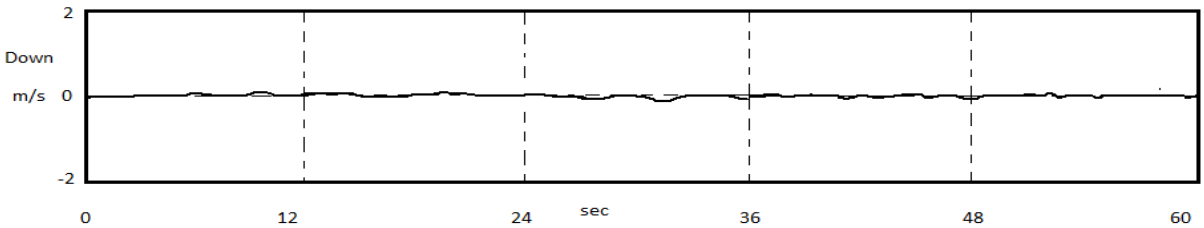

Fig7- Downward Velocity Error

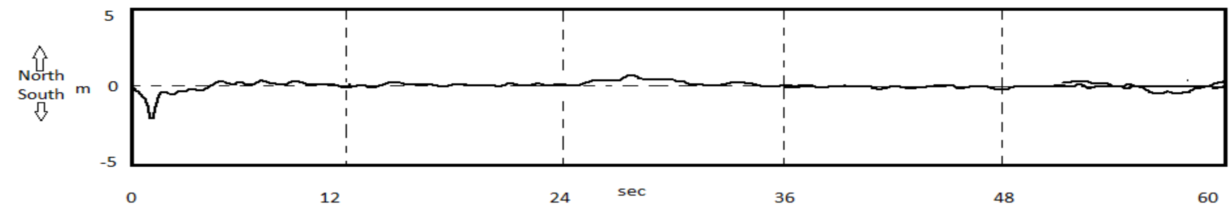

Fig8- North Position Error

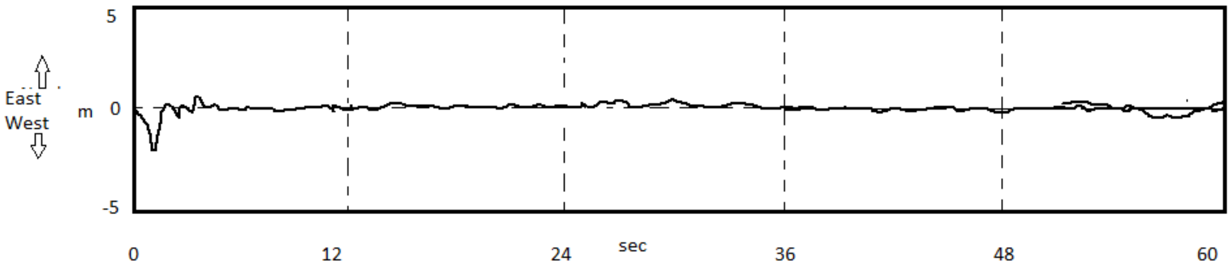

Fig9- East Position Error

Parameter	**Ref7**	**Ref8**	**Ref9**	**Observed value**
Mean squareerror	1.2780,1.1091	.04104,.04034,.03978	0.18 , 0.02	.0148,.0180,.0125,.0101
Error covariance	……..	………		.0163,.0201,.0149,.0121
Standard deviation	……..	………		.0115,.0161,.0109 ,.0091

Table 1-Parameter Values Obtained Through Simulation

VI.CONCLUSION

Mean square error, error covariance and standard deviation have been found as given in Table 1.The value of mean square error lies in the range from 0.01 to 0.018,value of error covariance lies in the range from 0.012 to 0.02 and the values of standard deviation lies in the range from 0.009 to 0.016.On the basis of above obtained results it can be seen that the tracking ability of the implemented method has been better as compared to the result given in [1],[2]and[3] in terms of accuracy and speed of convergence.however there are some portions in which certain dips and rises in the curve can be seen.The portion of time from 24s to 30s, the crests and 57 troughs signify a large value of error due to a change in the angular velocity and position of the satellite as a result of circular orbit of the satellite.

REFERENCES

[1]Zhou Donghua, Xi Yugeng, Zhang Zhongjun, "A Suboptimal Multiple Fading ExtendedKalman Filter" [J]. ActaAutomaticaSinica, 17(6), pp. 689-695, 1991.

[2] Sónia Maria Martinho Marques, "Small Satellites Attitude Determination Methods",Technical Report, Technical University of Lisbon, December 2000.

[3] Joshua Cemenska, "Sensor Modeling and Kalman Filtering Applied to Satellite Attitude Determination", Technical Report, University of California, 2004.

[4] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation", Proc. IEEE,Vol. 92, no. 3, pp. 401–422, Mar. 2004

[5] Maria Isabel Ribeiro, "Kalman and Extended Kalman Filters: Concept, Derivation and Properties",Technical Report, Institute for Systems and Robotics, February 2004

[6] R. Merwe, "Sigma Point Kalman Filters for Probabilistic Interference in Dynamic State- Space Models", Ph.D Thesis, OGI School of Science and Engineering at Oregon Health and Science University, 2004.

[7] RambabuKandepu, Bjarne Foss and Lars Imsland, "Applying the Unscented Kalman Filter for Nonlinear State Estimation", J. Process Contr. (2008), doi:10.1016/j.jprocont.2007.11.004.

[8] YanlingHao, ZhilanXiong, Feng Sun and XiaogangWang,"Comparison of KalmanFilters", In Proceedings of the 2007 IEEE International Conference on Mechatronics and Automation, Harbin, China, August 2007.

[9] Jaqueline M. Nelson, "Persistent Military Satellite Communications Coverage Using a CubeSatConstellation in Low Earth Orbit", Technical Report, University of Central Florida, 2010.

[10] Timothy Pratt, Charles Bostian and Jeremy Allnutt, "Satellite Communications", Second Edition, John Wiley and Sons, 2010.

[11] JongheeBae and Youdan Kim, "Attitude Estimation for Satellite Fault Tolerant System Using Federated Unscented Kalman Filter", International Journal of Aeronautical & Space Sciences 11(2), 80–86 (2010).

[12] Gerasimos G. Rigatos,"A Derivative Free Kalman Filtering Approach to State Estimation- Based Control of Nonlinear Systems",IEEE Transactions on Industrial Electronics,Vol.59, No. 10, pp. 3987-3997,Oct 2012.

[13] Dennis Roddy, "Satellite Communications", Fourth Edition, McGraw Hill, 2010.

[13] Hector Garcia de Marina,Felipe Espinosa and Carlos Santos, "Adaptive UAV Attitude Estimation Employing Unscented Kalman Filter, FOAM and Low-Cost MEMS Sensors",Sensors 2012, 12, 9566-9585; doi: 10.3390/s120709566.

[18] JacobaAuret, "Design of an Aerodynamic Attitude Control System for a CubeSat",Technical Report, University of Stellenbosch, March 2012

[16] Lubin Chang, Baiqing Hu, An Li, and Fangjun Qin, "Transformed Unscented KalmanFilter", IEEE Transactions on Automatic Control, Vol. 58, No. 1, pp. 252-257, January 2013

[19] Rudolph van der Merwe and Eric A. Wan,"The Square-Root Unscented Kalman Filter forState and Parameter-Estimation", Technical Report,Oregon Graduate Institute of Scienceand Technology, USA,feb 2013.

[20] AnkitAswal, AbhishekSaini and Lalit Singh Garia, "The Kalman Filter: Introduction andApplications", International Journal of Advances in Science and Technology, Vol. 6, No. 4,April 2013.

EXAMPLE FOLLOWS:

BOOKS:-

[1] Greg welch and Gary bishop,*An introduction to klaman filter*(Department of Computer Science Chapel Hill, NC 27599-3175)

[2] Eli brookner,tracking and kalman filtering made easy.

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