# QUEUING THEORY IN WORKSHOP

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**ABSTRACT**

*When we are out of the home, we see a huge number of vehicles in the street. It is observed that numbers of vehicles are increasing very rapidly. However, number of vehicles for repairing also increasing in the workshop. Now, there is a tough competition held between the workshops. When there is a huge waiting line in one workshop then a customer withdraw and go to the competitor's door; the service time may need to be improved. In this paper, we show that a queuing theory satisfies the model when tested with a real case scenario. A workshop in Arthoni provide us a data and also we derive the arrival rate, service rate, utilization rate, waiting time in queue and the probability of potential customers to balk based on the data using Little's theorem and M/M/1/GD/∞/∞ queuing model. The mean arrival rate at Kavisha during its busiest period of the day is 2.22 customers per minute (cpm) while the service rate is 2.24 cpm. The average number of customers in the workshop is 133 and the utilization period is 0.991. Our result involved the benefits of performing queuing analysis to a busy workshop.*

***Keywords — Queue, Little's Theorem, Workshop, Waiting Lines***

# I. INTRODUCTION

## 1.1 Workshop ( A Repair Shop)

The basic purpose of the workshop is to offer paintwork repairs to scratches, scuffs and dents to vehicle damage as well as damage caused by collisions and major accidents. Repair shops often can be specialty shops specializing in certain parts such as brakes, mufflers and exhaust systems, transmissions, body parts, tires, automobile electrification, automotive air conditioner repairs, automotive glass repairs and installation, and wheel alignment or those who only work on certain brands of vehicle or vehicles from certain continents of the world. There are also automotive repair shops that specialize in vehicle modifications and customization.

## 1.2 Need of the Queuing Theory in a Repair Shop

As the number of vehicles increases, number of workshop are not increasing as a result service time decreasing and customer became impatient and went to the competitor’s shop. There are several determining factors for a workshop to be considered a good or a bad one. Paint, Crash Parts, Refinishing Materials, Repair Materials, Tools, Capital Equipment, Mechanical Parts etc. are some of the most important factors. These factors, when managed carefully, will be able to attract plenty of customers. However, there is also another factor that needs to be considered especially when the workshop has already succeeded in attracting customers. This factor is the customers queuing time. Queuing theory is the study of queue or waiting lines. Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average time in the system, the expected queue length, the expected number of customers served at one time, the probability of balking customers, as well as the probability of the system to be in certain states, such as empty or full.

Waiting lines are a common sight in workshops especially during day and evening time. Hence queuing theory is suitable to be applied in a workshop setting since it has an associated queue or waiting line where customers who cannot be served immediately have to queue (wait) for service. Researchers have previously used queuing theory to model the restaurant operation [2], reduce cycle time in a busy fast food restaurant [3], as well as to increase throughput and efficiency [5].

This paper uses queuing theory to study the waiting lines in Kavisha Workshop in Arthoni, Agra. The workshop provides three workers per customer. There are 20 to 25 workers working at any one time. On a daily basis, it serves over 600 customers during weekdays, and over 2000 customers during weekends. This paper seeks to illustrate the usefulness of applying queuing theory in a real case simulation.

## 1.3 QUEUING THEORY

In 1908, Copenhagen Telephone Company requested Agner K. Erlang to work on the holding times in a telephone switch. He identified that the number of telephone holding time fit into Poisson distribution and exponentially distributed. This was the beginning of the study of queuing theory. In this section we will discuss two common concepts in queuing theory.

### 1.3.1 Little's Theorem

Little's theorem [7] describes the relationship between throughput rate (i.e. arrival and service rate), cycle time and work in process (i.e. number of customers/jobs in the system). This relationship has been shown to be valid for a wide class of queuing models. The theorem states that the expected number of customers (N) for a system in steady state can be determined using the following equation:

$$L = \lambda T \qquad (1)$$

Here, λ is the average customer arrival rate and T is the average service time for a customer. Consider the example of a workshop where the customer's arrival rate (λ) doubles but the customers still spend the same amount of time in the workshop (*T*). These facts will double the number of customers in the workshop (*L*). By the same logic, if the customer arrival rate (λ) remains the same but the customers service time doubles this will also double the total number of

customers in the workshop. This indicates that in order to control the three variables, managerial decisions are only required for any two of the three variables.

Three fundamental relationships can be derived from Little's theorem [6]:

- *L* increases if *λ* or *T* increases

- $\lambda$ increases if $L$ increases or $T$ decreases
- $T$ increases if $L$ increases or $\lambda$ decreases

Rust [8] said that the Little's theorem can be useful in quantifying the maximum achievable operational improvements and also to estimate the performance change when the system is modified.

### 1.3.2 Queuing Models and Kendall's Notation

In most cases, queuing models can be characterized by the following factors:

1. **Arrival time distribution:** Inter-arrival times most commonly fall into one of the following distribution patterns: a Poisson distribution, a Deterministic distribution, or a General distribution. However, inter-arrival times are most often assumed to be independent and memory less, which is the attributes of a Poisson distribution.

2. **Service time distribution:** The service time distribution can be constant, exponential, hyper-exponential, hypo-exponential or general. The service time is independent of the inter-arrival time.

3. **Number of servers:** The queuing calculations change depends on whether there is a single server or multiple servers for the queue. A single server queue has one server for the queue. This is the situation normally found in a grocery store where there is a line for each cashier. A multiple server queue corresponds to the situation in a bank in which a single line waits for the first of several tellers to become available.

4. **Queue Lengths (optional):** The queue in a system can be modeled as having infinite or finite queue length.

5. **System capacity (optional):** The maximum number of customers in a system can be from 1 up to infinity. This includes the customers waiting in the queue.

6. **Queuing discipline (optional):** There are several possibilities in terms of the sequence of customers to be served such as FIFO (First In First Out, i.e. in order of arrival), random order, LIFO (Last In First Out, i.e. the last one to come will be the first to be served), or priorities.

Kendall, in 1953, proposed a notation system to represent the six characteristics discussed above. The notation of a queue is written as:

$$A/B/P/Q/R/Z$$

where A, B, P, Q, R and Z describe the queuing system properties.

- A describes the distribution type of the inter arrival times.
- B describes the distribution type of the service times.
- P describes the number of servers in the system.
- Q (optional) describes the maximum length of the queue.
- R (optional) describes the size of the system population.
- Z (optional) describes the queuing discipline.

### 1.3.3 Maruti Workshop Queuing Model

The data were obtained from Kavisha workshop through interview with the workshop manager as well as data collections through observations at the restaurant. The daily number of visitors was obtained from the workshop itself. The workshop has been recording the data as part of its end of day routine. We also interviewed the workshop manager to find out about the capacity of the
workshop, the number of workers, as well as the number of machines in the workshop. Based on the interview with the workshop manager, we concluded that the queuing model that best illustrate the operation of Kavisha is M/M/1.
This means that the arrival and service time are exponentially distributed (Poisson process). The workshop system consists of only one server. In our observation the workshop has several workers but in the actual waiting queue, they only have one machine to serve all of the customers.

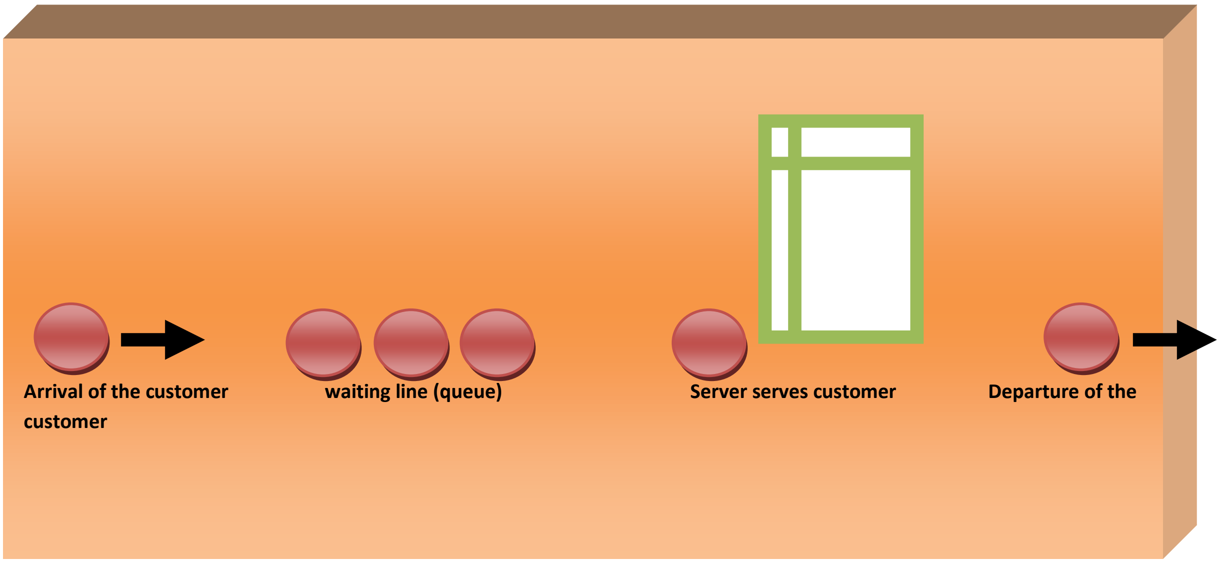


**Figure 1: M/M/1 Queuing Model**

For the analysis of the Kavisha M/M/1 queuing model, the following variables will be investigated [6]:

- **The mean customers arrival rate:** $\lambda$
- **The mean service rate:** $\mu$
- **Utilization factor:** $\rho = \lambda/\mu$ (1)
- **Probability of zero customers in the workshop:** $P_0 = 1 - \rho$ (2)
- **Probability of having *n* customers in the workshop:** $P_n = P_0\rho^n = (1-\rho)\rho^n$ (3)
- **Avera**

**ge number of customers served in the workshop:** $L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$ (4)

- **Average number in the queue:** $L_q = L \times \rho = \frac{\rho^2}{1-\rho} = \frac{\rho\lambda}{\mu-\lambda}$ (5)
- **Average time spent in Kavisha, including the waiting time:** $W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda}$ (6)
- **Avera**

**ge waiting time in the queue:** $W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu-\lambda}$ (7)

## II. RESULT AND DISCUSSION

The one month daily customer data were shared by the workshop manager as shown in Table I.

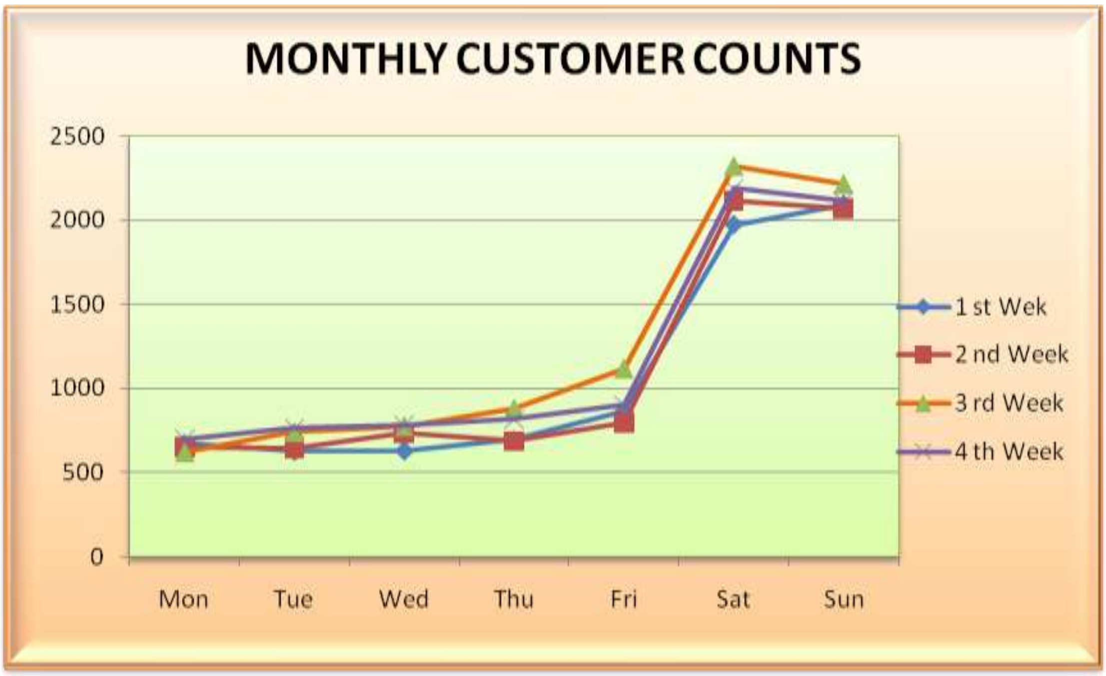


**Figure 2: One Month daily customer counts**

As can be seen in Figure 2, the number of customers on Saturdays and Sundays are double the number of customers during weekdays. The busiest period for the workshop is on weekend during day time. Hence, we will focus our analysis in this time window.

### 2.1 . Calculation

Our teams conducted the research at dinner time. There are on average 600 people are coming to the workshop in every 270 minutes time window of day time. From this we can derive the arrival rate as:

$$\lambda = \frac{600}{270} = 2.22 \; customers\,/minute\,(cpm)$$

We also found out from observation and discussion with manager that each customer spends one hour on average in the workshop (*W*), the queue length is around 40 people (*Lq*) on average and the waiting time is around 18 minutes.

It can be shown using (7) that the observed actual waiting time does not differ by much when compared to the theoretical waiting time as shown below.

$$W_q = \frac{40 \; customers}{2.22 \; cpm} = 18.02 \; minutes$$

Next, we will calculate the average number of people in the workshop using (1).

$$L = 2.22 \; cpm \; \times 60 \; mins. = 133.2 \; customers$$

Having calculated the average number of customers in the workshop, we can also derive the utilization rate and the service rate using (4).

$$\mu = \frac{\lambda(1+L)}{L} = \frac{2.22(1+133.2)}{133.2} = 2.24\ cpm$$

$$Hence, \rho = \frac{\lambda}{\mu} = \frac{2.22\ cpm}{2.24\ cpm} = 0.991$$

With the very high utilization rate of 0.991 during day time, the probability of zero customers in the workshop is very small as can be derived using (2).

$$P_0 = 1 - \rho = 0.019$$

The generic formula that can be used to calculate the probability of having *n* customers in the workshop is as follows:

$$P_n = (1 - 0.991)0.991^n = 0.019(0.991)^n$$

We assume that potential customers will start to balk when they see more than 10 people are already queuing for the workshop. We also assume that the maximum queue length that a potential customer can tolerate is 45 people. As the capacity of the workshop when fully occupied is 130 people, we can calculate the probability of 10 people in the queue as the probability when there are 140 people in the system (i.e. 130 in the restaurant and 10 or more queuing) as follows:

*Probability of customers going away = P (more than 15people in the queue) = P (more than 140 people in the workshop)*

### 2.2. Evaluation

1. The utilization is directly proportional with the mean number of customers. It means that the mean number of customers will increase as the utilization increases.
2. The utilization rate at the workshop is very high at 0.991. This, however, is only the utilization rate during day time on Saturdays and Sundays. On weekday, the utilization rate is almost half of it. This is because the number of visitors on weekdays is only half of the number of visitors on weekends. In addition, the number of workers or machines remains the same regardless whether it is peak hours or off-peak hours.
3. In case the customers waiting time is lower or in other words we waited for less than 15 minutes, the number of customers that are able to be served per minute will increase. When the service rate is higher the utilization will be lower, which makes the probability of the customers going away decreases.

### 2.3 Benefits

1. This research can help Kavisha to increase their QOS (Quality Of Service), by anticipating if there are many customers in the queue.
2. The result of this paper work may become the reference to analyze the current system and improve the next system. Because the workshop can now estimate of how many customers will wait in the queue and the number of customers that will go away each day.

3. By anticipating the huge number of customers coming and going in a day, the workshop can set a target profit that should be achieved daily.
4. The formulas that were used during the completion of the research is applicable for future research and also could be use to develop more complex theories.
5. The formulas provide mechanism to model the workshop queue that is simpler than the creation of simulation model in [9,4].

## III. CONCLUSION

This research paper has discussed the application of queuing theory of Kavisha Workshop. Here we have focused on two particularly common decision variables (as a vehicle for introducing and illustrating all the concepts. From the result we have obtained that the rate at which customers arrive in the queuing system is 2.22 customers per minute and the service rate is 2.24 customers per minute. The probability of buffer flow if there are 10 or more customers in the queue is 15 out of 100 potential customers. The probability of buffer overflow is the probability that customers will run away, because may be they are impatient to wait in the queue. This theory is also applicable for the workshop if they want to calculate all the data daily. It can be concluded that the arrival rate will be lesser and the service rate will be greater if it is on weekdays since the average number of customers is less as compared to those on weekends. The constraints that were faced for the completion of this research were the inaccuracy of result since some of the data that we use was just based on assumption or approximation. We hope that this research can contribute to the betterment of Kavisha Workshop in terms of its way of dealing with customers.

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