# GOLDEN RATIO: THE RATIO OF BEAUTY

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## ABSTRACT

*This paper deals with Golden Ratio. The relationship between Golden Ratio and Fibonacci sequence is described. In ancient time, Golden Ratio was used by many mathematicians, artists and architects. Nature and our body also follow Golden Ratio. In these days we are using Golden Ratio in designing logos, magazines covers, to click pictures perfectly and in to give proper shape to our face in surgery.*

*Keywords: Fibonacci sequence, Golden Ratio*

## I. INTRODUCTION

When we talk about beauty- paintings, architectures, humans, nature's beautiful creations come in our mind. Paintings, sketches, designs of architectures are all associated with drawings. Beauty of Mathematics is that like other subjects, it has a deep relationship with drawing. Drawing and Mathematics go side by side. In drawing any picture, model, portrait or design or to design a building one need the knowledge of Mathematics in forms of geometrical forms and skills as well as arithmetical measurement and computation. [1] Though meaning of beauty varies from person to person, but Mathematics has a ratio to measure beauty called 'Golden Ratio'.

### 1.1 Golden Ratio

Golden Ratio is also known as Golden Mean, Golden Section, Divine Number which is denoted by Greek letter Phi. What is Golden Ratio? The Golden Ratio is the number 1.61803399. This is the number which was written by Euclid in his book 'Elements' around 300 BC. Luca Pacioli wrote about it in 'De Divine          Proportion' in 1509. Luca Pacioli was contemporary to Leonardo Da Vinci. In 1600, Johannes Kepler and in 2003 Dan Brown wrote about Golden Ratio in his novel ' The Da Vinci Code'.[2]

According to Mario Livio

"Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid in ancient Greece  through the medieval Italian mathematician Leonardo of Pisa and the Renaissance astronomer Johannes Kepler, to present-day scientific figures such as Oxford physicist Roger Penrose, have spent endless hours over this simple ratio and its properties. But the fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics."[3]

### 1.2  How To Calculate Golden Ratio

Golden ratio exists when a line is divided into two parts and the longer part (a) divided by the smaller part (b) is equal to the sum of (a) + (b) divided by (a), which both equal 1.6180339887
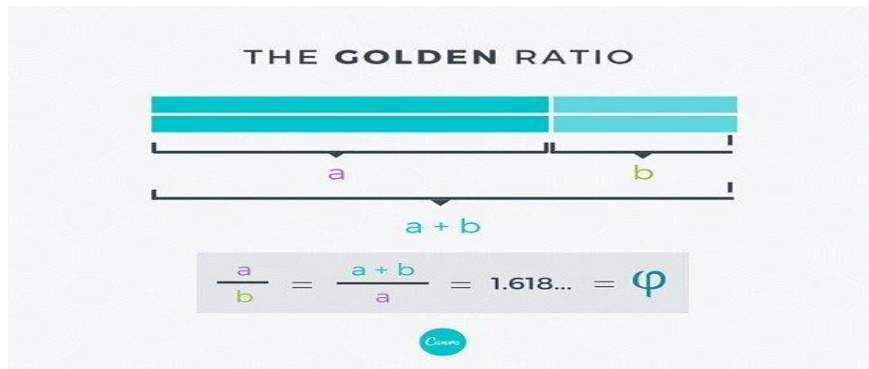
**Figure 1**

The Golden Ratio can be applied to shapes too. Take a square and multiply one side of by 1.618 and we get a rectangle of harmonious proportions.  We lay the square over the rectangle the two shapes will give us the Golden Ratio:



**Figure 2**

These rectangles are called golden rectangles. By adding the arch in each square, we'll end up with the diagrams
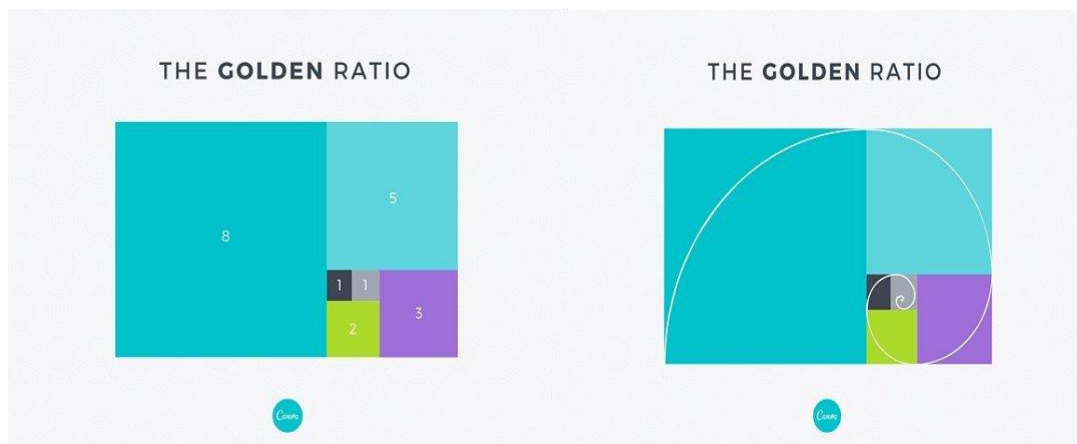


Figure 3

The other method to describe Golden Rectangles is: if we have a rectangle, then take its smaller side and make a square inside that rectangle then in bigger rectangle we will have a square and a smaller rectangle. Again make a square in smaller rectangle, then in smaller rectangle we will left with a rectangle and square  continuing like this we will have a figure like figure 3.  [4]

## II. RELATIONSHIP BETWEEN FIBONACCI SEQUENCE AND GOLDEN RATIO

Fibonacci sequence is named after Italian Mathematician Leonardo Pisano ( known as Fibonacci worldwide). Fibonacci used the arithmetic series to illustrate a problem based on a pair of breeding rabbits. The problem is described as – There is a pair of male and female rabbit. How many pair of rabbits could be born in a year? The conditions of this problem are:

1. Begin with one male rabbit and female rabbit that have just been born.
2. Rabbits reach sexual maturity after one month.
3. The gestation period of a rabbit is one month.
4. After reaching sexual maturity, female rabbits give birth every month.
5. A female rabbit gives birth to one male rabbit and one female rabbit.
6. Rabbits do not die.

The production of rabbits can be expressed by a sequence 1,1,2,3,5,8,13,21,34,.............. which was given by Fibonacci. This sequence is obtained by adding two last numbers and next number is obtained i.e. 1+1=2, 1+2=3, 2+3=5, 3+5=8 and so on. . The Fibonacci sequence can be expressed as recurrence formula which is $F(n) = F(n-1) + F(n-2)$ given $F(0)= 1, F(1)= 1$. ( add the previous two to obtain next). [5]

Fibonacci sequence and Golden Ratio have very close connection. The Fibonacci sequence is 1, 1,2,3,5,8,13,21,34,...........  If we take two successive numbers and divide the larger number with the smaller one then the number, which we will obtain, is very close to Golden Ratio i.e 1.618034.. the more larger number we take, the ratio will go more close to Golden Ratio. For example: 3/2= 1.5 , 5/3= 1.6666 , 8/5= 1.6, 34/21=1.61904 ..................... 377/233= 1.618025 and so on. This rule also works for if we start Fibonacci sequence from any two random numbers.  [6] . Also there is a fact that from golden ratio we can also find Fibonacci Numbers. The formula to find Fibonacci Number from Golden Ratio is  $x(n) = \Phi^n – (1-\Phi)^n / (5)^{1/2}$

Put n= 4, $x(4)= (1.6180)^4 - (0.6180)^4 / (5)^{1/2}$ = 6.8535 -0.09014/2.2360 = 3.024. The more accurate value of $\Phi$ will give more precise value which will be 3, which is on the fourth place of sequence 1,1,2,3,5,8,,,,,,.

## III. USE OF GOLDEN RATION IN HISTORY

3.1 The Vitruvian Man of Leonardo da Vinci :  Vitruvian man was created by Leonardo Da Vinci in 1500 AD. The Vitruvian Man illustrates a variety of different proportions in the human body. With arms outstretched, a man is as wide as he is tall, with the genitals as the midpoint. (Green). The knees are halfway between the genitals and the feet. (Blue). The chest is halfway between the genitals and the top of the head. (Blue). The chest is the same width as a quarter of the height. (Red). The measurement from the elbow to the fingertips is also a quarter of the height (Red). The nose is halfway between the hairline and the chin. (Orange). The eyebrows are halfway between the nose and hairline. (Orange). The lips are halfway between the nose and chin. (Orange). The head is one-eighth the total height. The hand is one-tenth the total height. The foot is one-sixth of the total eight.
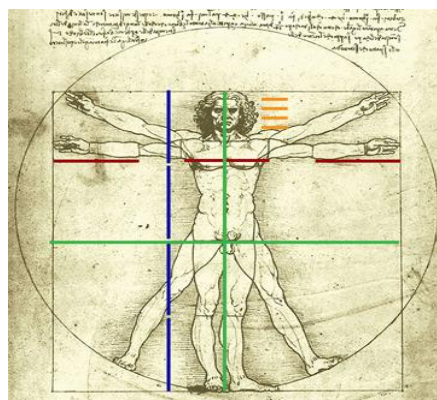
**Figure 4**

This figure explains when a man lie on his back and extend his arms, his hands and feet will touch the circle with centre at his navel . The hands and feet will touch the sqaure when arms are just streched horizontly. The ratio of foot to navel and navel to head is golden ratio.[7]

3.2 Pyramid of Giza: The number can be seen in the architecture of many ancient creations, like the Great Pyramids. In the Great Pyramid of Giza, which was built in 2589-2504 BC, the length of each side of the base is 756 feet with a height of 481 feet. The ratio of the base to the height is roughly 1.5717, which is close to the Golden ratio.

3.3 Parthenon: Phidias (500 BC – 432 B.C.) was a greek sculptor and mathematician who is thought to have applied golden ratio to the design of the Parthenon. [8]

3.4 Mona Lisa ( figure 5), painted by Leonardo Da Vinci, is drawn according to the golden ratio. The Mona Lisa has many golden rectangles throughout the painting. By drawing a rectangle around her face, we can see that it is indeed golden. If we divide that rectangle with a line drawn across her eyes, we get another golden rectangle, meaning that the proportion of her head length to her eyes is golden. There are other golden rectangles that can be drawn on the rest of her body, like from her neck to the top of her hands. [9]

3.5 Last Supper: Leonardo Da Vinci used golden ratio to define all the fundamental proportions of his painting of "The Last Supper," (figure 6) from the dimensions of the table at which Christ and the disciples sat to the proportions of the walls and windows in the background sculptures to achieve balance and beauty.[10]
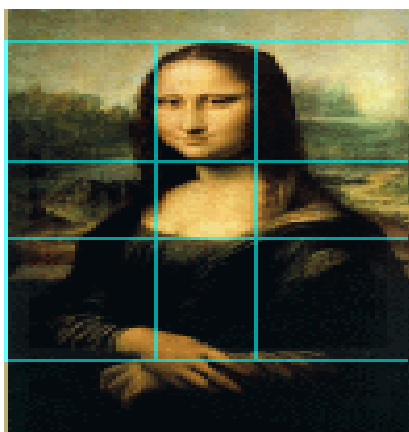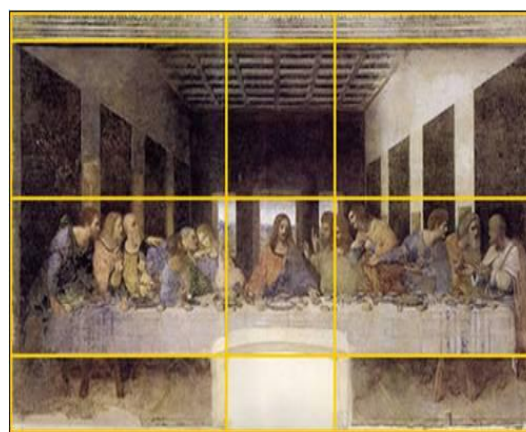


**Fig.5**



**Fig.6**

3.6 Plato ( 428 B.C – 347 B.C) considered the golden ratio to the most universally building of mathematical relationships. Euclid (365 B.C – 300 .B.C) linked golden ratio to the construction of pentagram.

## IV GOLDEN RATIO AND NATURE

Not only the ancient architects, but the nature itself follows Golden Ratio. Flowers , sea shells, honeycombs, Pineapples and huricanes and many more explicit Golden Ratio in their designs
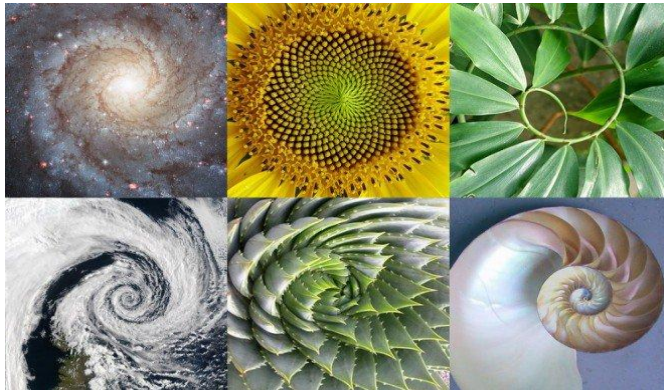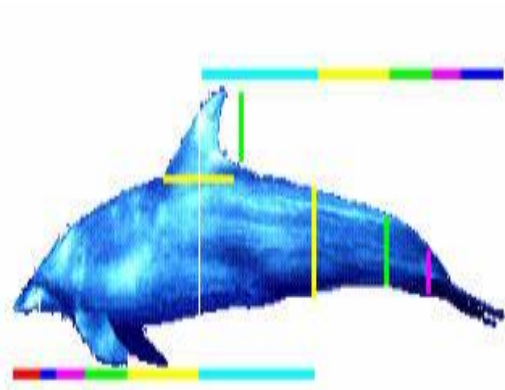


**Figure 7**



**Figure 8**

All these are example of sprials which can be obtained from sprials made by golden rectangles. The eyes, fins and tail of the dolphin fall at Golden sections along the body. (Figure 8)

Even our body and faces follows golden ratio. The ratio between the forearm and the hand is the Golden Ratio In these days, plastic surgens follows golden ratio for surgeries.
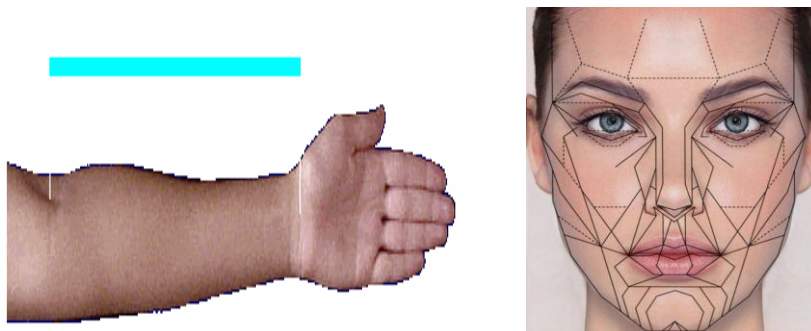


**Figure 9**

## V. MODERN USE OF GOLDEN RATIO

Golden Ratio is like gold for designers in present time. The graphic designers are using Golden Ratio and Golden Spiral to enhance their designs or work. Many website has been designed with the help of Golden Ratio or Golden Spiral or Golden Rectangle or golden Circle. National Geographic's website, Terkaya Weath Managment's visual identity , visual identity of Saastamoisen Säätiö, the logo of Fullsteam Brewery follow golden rectangle and golden spiral. The Pepsi logo is also based on two circles that follow the golden ratio. The Twitter logo uses geometry and is heavily based on perfect circles. There is minor lack of precision when

aligning it with the Golden Ratio but for the most part the Twitter logo uses Golden circles for balance, order and harmony.[ 11]



**Figure 10**

## VI.  PHIMATRIX- SOFTWARE OF GOLDEN RATIO

Gary Meisner is developer of PhiMatrix and he is also the author of Phi: The Golden Number. The concept of PhiMatrix was developed in 1997 and PhiMatrix software was developed in 2004. This software helps to appreciate the beauty and design in the world around us and also apply the same principle ,found in nature, in our own creative pursuits. PhiMatrix apply the mathematics, trigonometry and geometry in nature with ease of selecting a template and clicking and dragging it over anything we desire to design. This software make us to understand limitless application of Golden Ratio in Logos, Graphics, Photographs, Art, Architecture and many more. [12]

## VII.  CONCLUSION

Golden Ratio attracted the mathematician, architects and painters of ancient time. They used it to create beautiful geometrical figures, monuments or paintings. In present era software developers, mathematicians, designers, plastic surgeons has taken it to its new heights which are good mathematics and human being both.

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