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STRESS ANALYSIS OF MESHING GEARS WITH ADDENDUM MODIFICATION

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ABSTRACT

A study into the effects of addendum modification on root stress in involute spur gears with various pressure angles, teeth and modules are presented in this project work. The range of addendum modification co-efficient is taken from negative value to positive value through zero by considering both the upper limit (peaking limit) and the lower limit (undercutting limit). The root stress factor is found out for various loading positions. The variation of root stress factor with addendum modification co-efficient is shown when only the driving gear is modified. A study into the effects of addendum modification on root stress using mathematical formulation as well as finite element analysis when both the driver and follower are modified at the same time also for different gear ratios. The value of root stress factor decreases with an increasing addendum modification coefficient when only the driver is modified. The root stress factor also decreases when pressure angle is increased. The root stress factor is further decreased when the module and number of teeth are increased. Analysis on Ansys and theoretical analysis both confirm the results and were found close to each other.

Keywords : Addendum modification coefficient, Root Stress, Tooth Thickness.

I. INTRODUCTION

The increasing demand for high tooth strength and high load carrying capacity of gears leads to various methods of improvements. One of the major method available till now is "Profile Shift". In gear technology it is known as "Addendum Modification". The amount by which the addendum is increased or decreased is known as "Addendum Modification". The aim of addendum modification is to avoid interference. Previously various methods used to avoid interference were: Undercutting at the root, Making the mating gear tooth stub, Using a minimum number of teeth in a gear for a certain pressure angle. But undercutting weakens the tooth strength severely and there may be the situation where a smaller number of teeth in a gear is to be adopted. But addendum modification avoids all these difficulties. It is also known that profile-shifted gears as compared to standard gears, offer a lot of advantages. The load carrying capacity of the gears can be greatly improved without

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any appreciable change in gear dimensions by adopting addendum modification. Now-a-days profile shifted gears are more often due to its reduced vibration and reduced noise property. In this project work the effects of addendum modification on root stress are investigated. In this project report, influence of modification of addendum of involute spur gear tooth on its root stress has been studied for different pressure angles, teeth and modules.

II. ROOT STRESS FORMULAS AND DIMENSIONS FOR CALCULATIONS[1][2]

2.1 Dimensions required for calculation of Root stress

When a rack tooth is used for generating gear teeth, the form of fillet of the generated gear tooth becomes a trochoid curve. Taking the co-ordinate system and symbols for the trochoid curve as shown in Fig.1, the co-ordinates (X,Y) on the fillet curve at the root of the gear tooth are derived as follows:

$$X = (R-h_0)\sin(\varphi+u) - R\varphi\cos(\varphi+u) - \frac{\{R\varphi\cos(\varphi+u)+h_0\sin(\varphi+u)\}r_0}{\sqrt{h_0^2+R^2\varphi^2}} \quad ...(1)$$

$$Y = (R-h_0)\cos(\varphi+u) + R\varphi\sin(\varphi+u) - \frac{\{R\varphi\sin(\varphi+u)-h_0\cos(\varphi+u)\}r_0}{\sqrt{h_0^2+R^2\varphi^2}} \quad ...(2)$$

When θ is angle between centerline of gear and tangent to fillet curve

$$tan\theta = -\frac{dX}{dY} = -\frac{dX/d\gamma}{dY/d\gamma} = \frac{h_0 - R\varphi\tan(\varphi+u)}{h_0\tan(\varphi+u)+R\varphi} \quad ...(3)$$

h_o = Distance from center line to center of rounded corner of rack tooth = $(1-x)m - r_0 sin\alpha_1$ ...(4)

r_o = Edge radius of generating rack = $\frac{C_k}{1-sin\alpha_1}$ ...(5)

l_o = Distance from centerline of gear tooth to foot of perpendicular = $\frac{\pi m}{4} + h_0 tan\alpha_1 + r_0 sec\alpha_1 + xm tan\alpha_1$

from center of rounded corner of rack tooth on pitch line ...(6)

u = Angle around center of gear = $\frac{l_0}{R}$

ρ = Radius of curvature of fillet curve at critical section = $\frac{(h_0^2+R^2\varphi^2)^{3/2}}{R^2\varphi^2+Rh_0+h_0^2} + r_0$ ...(7)

S = Tooth thickness at critical section = 2X

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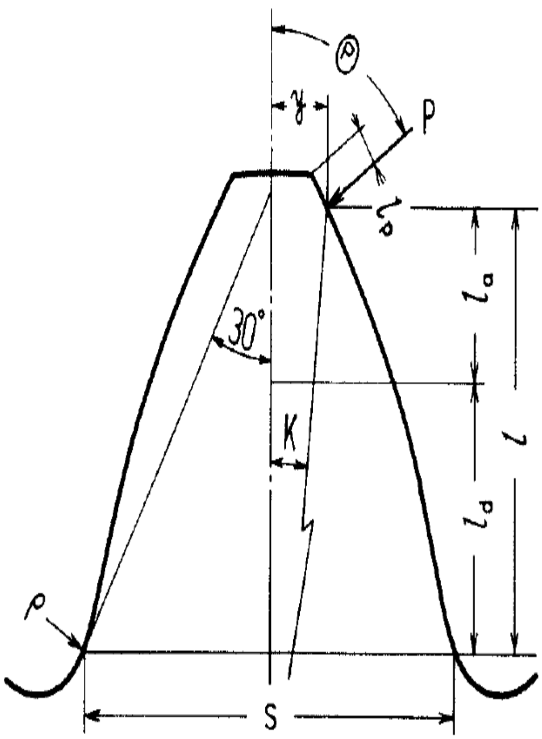
l_d = Distance from pitch circle to critical section = R – Y

ⓟ = Angle between direction of loading and centerline of tooth = $\frac{\pi}{2} + K - \alpha_p$ ...(8)

K = Angle between line connecting loading point with center = $\frac{\pi + 4x tan\alpha_1}{2Z} + inv\alpha_1 - inv\alpha_p$ ...(9)

of gear and centerline of gear tooth

l_a = Distance from loading point to pitch circle = $\frac{mZ}{2}(\frac{cos\alpha_1}{cos\alpha_p}cosK - 1)$ ...(10)



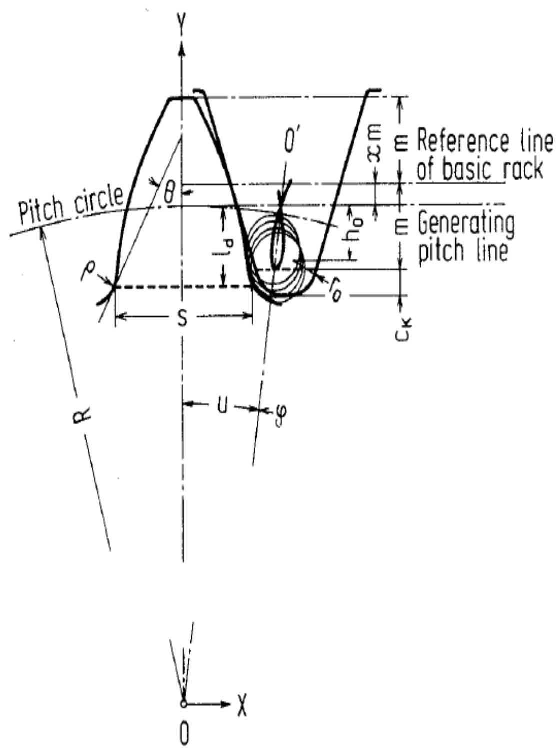


Fig.1 Dimensions for calculation of Root stress[1] **Fig.2 Root fillet curve and co-ordinate system[1]**

l = Distance from loading point to critical section = $l_a + l_d$

α_k = Pressure angle at the tip of gear tooth = $cos^{-1}(\frac{Zcos\alpha_1}{Z+2(1+x)})$ ...(11)

α_p = Pressure angle at loading point = $tan^{-1}tan^2\alpha_k - 2\frac{l_p}{R_b}$ ...(12)

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Where l_p = Distance from loading point to tip along tooth profile

R_b = Base circle radius of gear

y = Distance from loading point to center line of gear tooth = $\frac{mZ}{2}\frac{cos\alpha_1}{cos\alpha_p}sinK$ ...(13)

φ = Rolling angle of rack

2.2 Formula for true stresses at root fillet

True stresses at the root fillet of the gear tooth are calculated from the following formula:

$$\sigma_t = A\frac{P}{b}$$ ...(14)

Where A : Root stress factor for calculating stresses at root fillet given by eqn. (15)

$$A = \left(1 + 0.08\frac{S}{\rho}\right)(0.66A_b + 0.40\sqrt{A_b^2 + 36A_\tau^2} + 1.15A_c$$ ...(15)

Values of A_b,A_τ and A_c in eqn. (15) are given by eqn. (16)

Where $A_b = \frac{6lsin®}{S^2}$, $A_\tau = \frac{sin®}{S}$, $A_c = -\frac{cos®}{S} - \frac{6ycos®}{S^2}$ ...(16)

III. EXPERIMENTAL ANALYSIS

3.1 Design on Solidworks

The entire gear pair for experimental analysis is designed in solidworks using equations. All the parameters like involute profile, base radius, pitch radius, tip radius etc. are defined by equations in terms of pressure angle, module and number of teeth. The change in module, pressure angle or number of teeth instantly changes the whole gear structure according to the parameters provided. The solidworks model and equations used for designing it are shown as follows:

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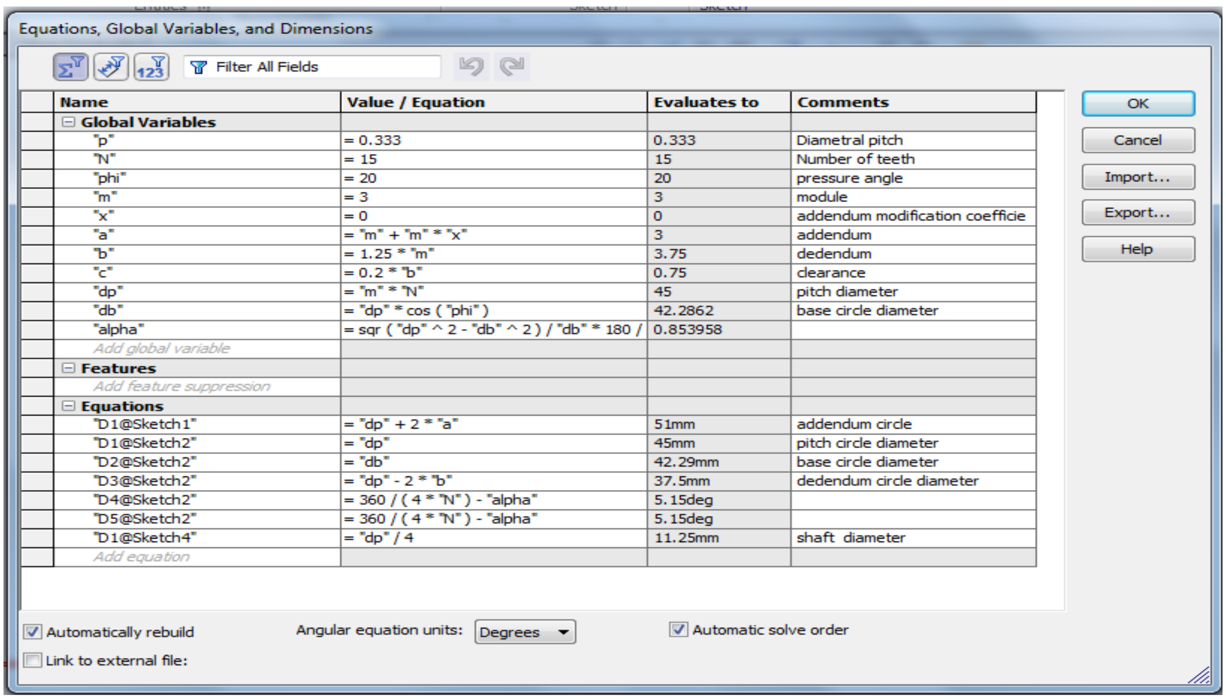

Equations, Global Variables, and Dimensions

Filter All Fields

Name	Value / Equation	Evaluates to	Comments
Global Variables			
"p"	= 0.333	0.333	Diametral pitch
"N"	= 15	15	Number of teeth
"phi"	= 20	20	pressure angle
"m"	= 3	3	module
"x"	= 0	0	addendum modification coefficie
"a"	= "m" + "m" * "x"	3	addendum
"b"	= 1.25 * "m"	3.75	dedendum
"c"	= 0.2 * "b"	0.75	clearance
"dp"	= "m" * "N"	45	pitch diameter
"db"	= "dp" * cos ("phi")	42.2862	base circle diameter
"alpha"	= sqr ("dp" ^ 2 - "db" ^ 2) / "db" * 180 /	0.853958	
Add global variable			
Features			
Add feature suppression			
Equations			
"D1@Sketch1"	= "dp" + 2 * "a"	51mm	addendum circle
"D1@Sketch2"	= "dp"	45mm	pitch circle diameter
"D2@Sketch2"	= "db"	42.29mm	base circle diameter
"D3@Sketch2"	= "dp" - 2 * "b"	37.5mm	dedendum circle diameter
"D4@Sketch2"	= 360 / (4 * "N") - "alpha"	5.15deg	
"D5@Sketch2"	= 360 / (4 * "N") - "alpha"	5.15deg	
"D1@Sketch4"	= "dp" / 4	11.25mm	shaft diameter
Add equation			

OK
Cancel
Import...
Export...
Help

Automatically rebuild
Angular equation units: Degrees
Automatic solve order
Link to external file:

Fig.3 Equations used in solidworks model

The equations used for drawing of involute profile in solidworks were as follows:

$$X_t = "D2@Sketch2" * 0.5 * (cos(t) + t * sin(t))$$

$$Y_t = "D2@Sketch2" * 0.5 * (sin(t) - t * cos(t))$$

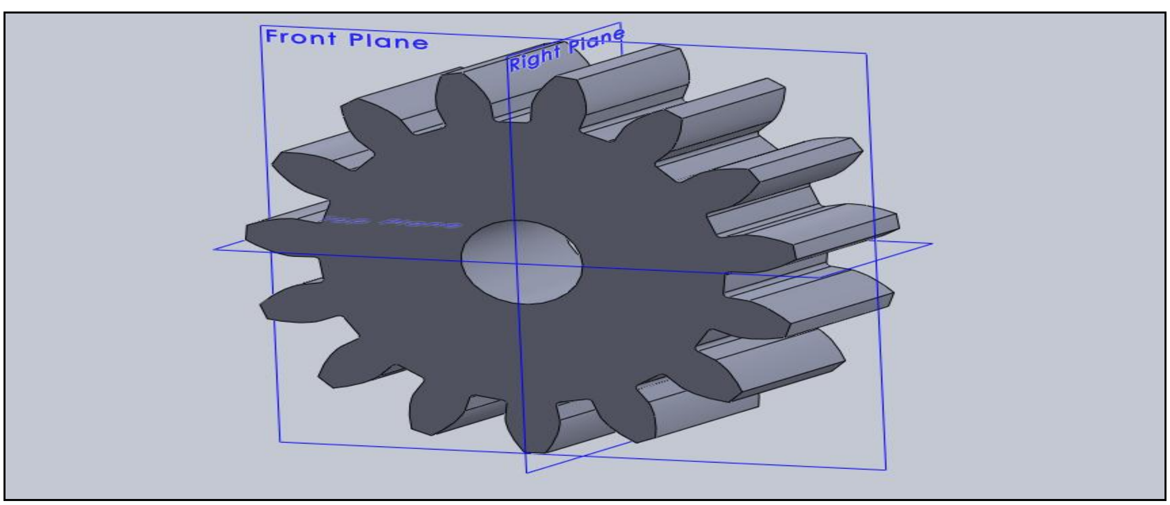

Fig. 4 Solidworks model of gear

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3.2 Analysis in Ansys Workbench

For analysis in workbench, application is considered as a centrifugal pump. The pinion speed is taken as 3000 rpm and gear ratio is taken as 3 everywhere. The sum of addendum modification coefficients of gear and pinion is taken as 0. Accordingly, corresponding gears for pinions are designed by varying the parameters in the equations mentioned above. The models are tested for different addendum modification coefficients with varying teeth, pressure angles and modules. The models are tested for stress and deformation. The workbench result is shown below:

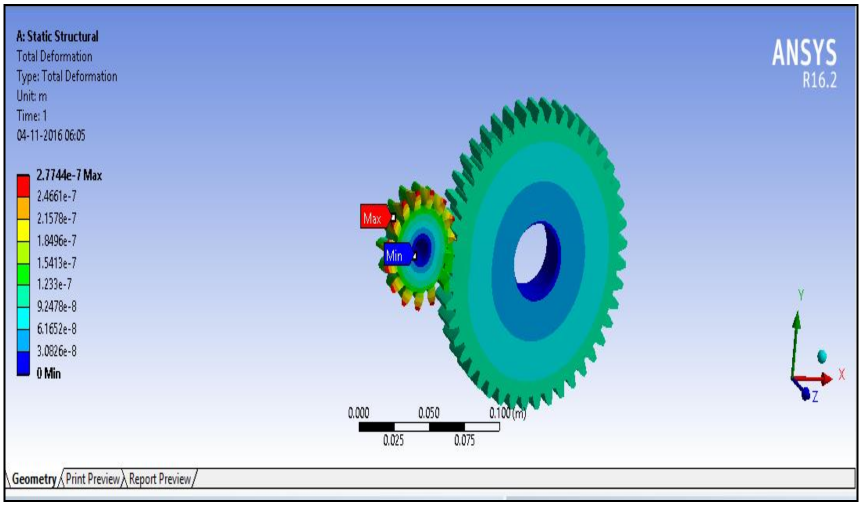

Fig. 5 Analysis showing total deformation

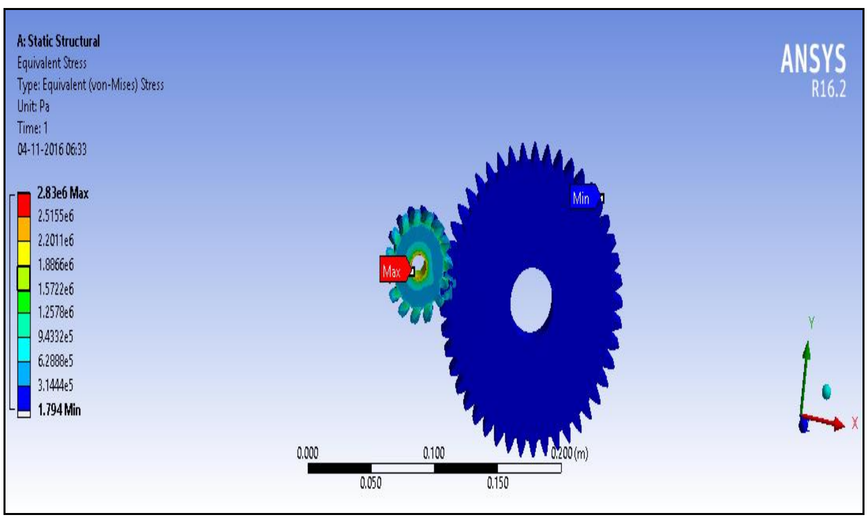

Fig.6 Analysis showing equivalent stress

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After reviewing the results of Ansys, following observations were made:

- As number of teeth increases, the root stress decreases for different values of addendum modification coefficients from negative value to positive value i.e. from -0.4 to 0.4. The deformation and stress is higher for negative values of addendum modification coefficients as compared to higher value.
- As the pressure angle is varied from 14 degrees to 20 degrees, the stress and deformation decreases with increase in pressure angle. Stress and deformation also show a decreasing trend with increase in addendum modification coefficients from negative to positive value.
- As the module is varied from 3 to 6, the root stress as well as deformation shows a decrease with increase in module. Also, there is a decrease in stress and deformation with increase in values of addendum modification coefficient from negative to positive value.

IV. RESULTS FROM THEORETICAL FORMULAE

The equations from (1) to (17) are used to find the root stress and tooth thickness for different values of addendum modification coefficient for different pressure angles, modules, and teeth.

4.1 Variation in pressure angle

The root stress and tooth thickness are plotted against addendum modification coefficient for different pressure angles. The tabular results and graphs are as follows:

Table 1. Root stress vs. addendum modification coefficient for varying pressure angles

Addendum modification coefficient	Root stress σ_t			
	Φ = 14	Φ = 16	Φ = 18	Φ = 20
-0.4	2.281204	2.271515	2.256782	2.237240
-0.3	1.986018	1.979325	1.968525	1.953760
-0.2	1.756763	1.752019	1.743839	1.732301
-0.1	1.573745	1.570303	1.563919	1.554628
0	1.424315	1.421758	1.416633	1.408945
0.1	1.300001	1.298053	1.293827	1.287302
0.2	1.194933	1.193406	1.189828	1.184162
0.3	1.104918	1.103682	1.100576	1.095552
0.4	1.026891	1.025855	1.023094	1.018551

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Table 2. Tooth thickness vs. addendum modification coefficient for varying pressure angles

Addendum modification coefficient	Tooth thickness S			
	Φ = 14	Φ = 16	Φ = 18	Φ = 20
-0.4	4.36467	4.38456	4.4061	4.42956
-0.3	4.87609	4.89695	4.91956	4.94417
-0.2	5.38784	5.40969	5.43338	5.45918
-0.1	5.89993	5.92278	5.94757	5.97456
0	6.41234	6.43622	6.46211	6.49031
0.1	6.92507	6.94998	6.977	7.00643
0.2	7.43811	7.46407	7.49222	7.52289
0.3	7.95145	7.97846	8.00777	8.0397
0.4	8.46508	8.49317	8.52364	8.55683

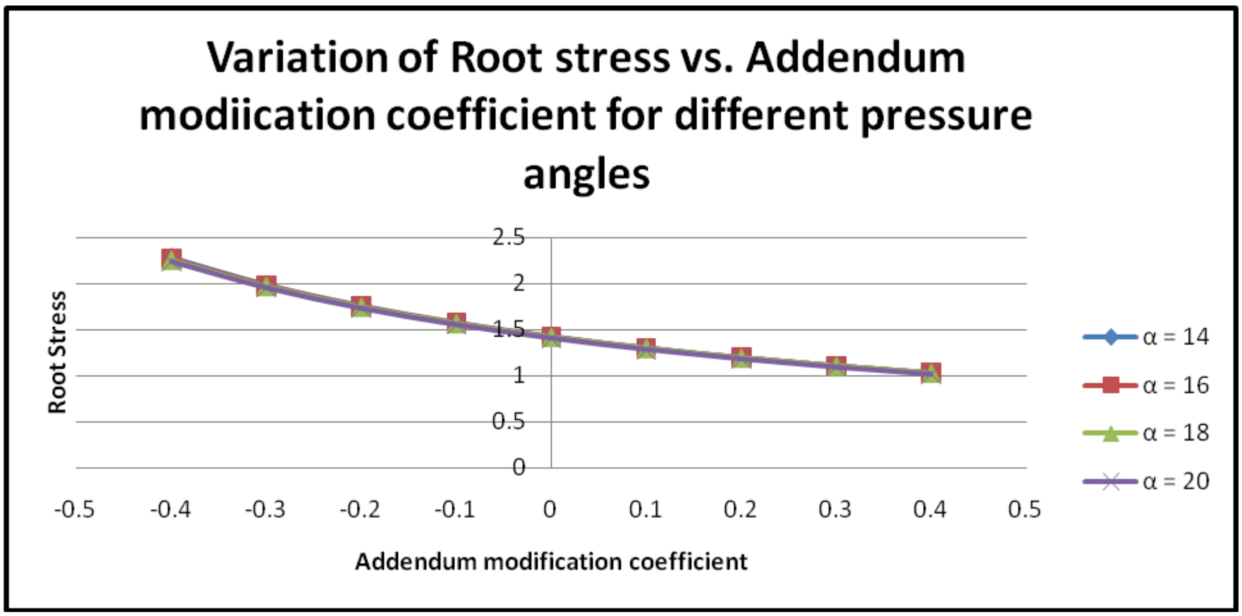

Fig. 7 Graphical representation of root stress vs. addendum modification coefficient

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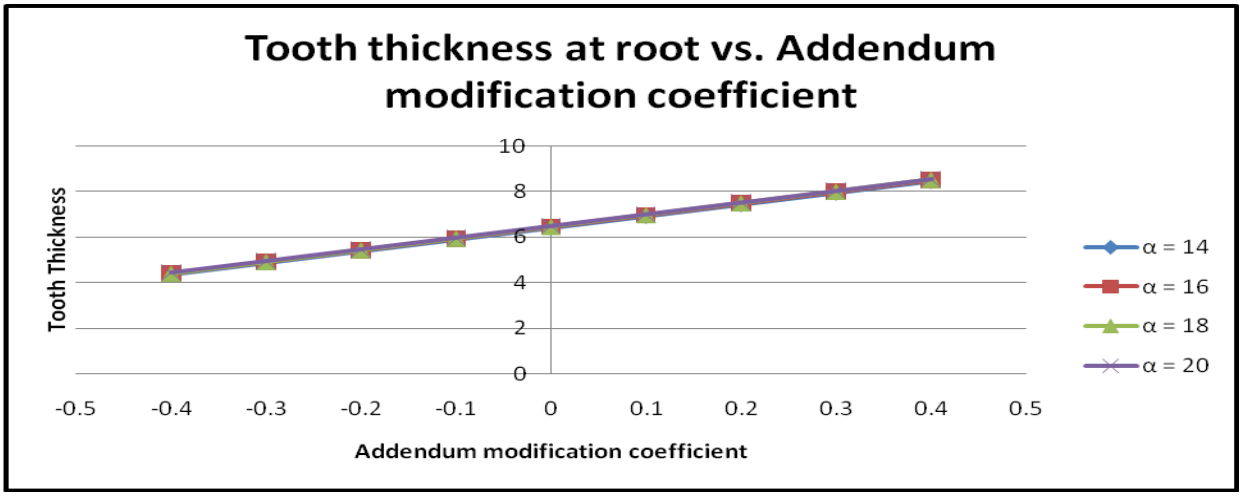

Fig. 8 Graphical representation of tooth thickness vs. addendum modification coefficient

Graphical representation shows that root stress decreases with increase in addendum modification coefficient. The root stress also decreases with increase in pressure angle. The tooth thickness also increases with increases with increase in addendum modification coefficient.

4.2 Variation in number of teeth

The root stress and tooth thickness are plotted against different values of addendum modification coefficients for varying number of teeth. The tabular results and graphs are as follows:

Table 3. Root stress vs. addendum modification coefficient for varying teeth

Addendum modification coefficient	Root stress σ_t			
	Z = 15	Z = 16	Z = 17	Z = 18
-0.4	2.23724	1.842944	1.568058	1.36551
-0.3	1.95376	1.646817	1.424329	1.255614
-0.2	1.732301	1.487452	1.304139	1.161692
-0.1	1.554628	1.355422	1.20214	1.080489
0	1.408945	1.244237	1.114475	1.009569
0.1	1.287302	1.149295	1.038297	0.947073
0.2	1.184162	1.067242	0.971458	0.891563
0.3	1.095552	0.995582	0.91231	0.841908
0.4	1.018551	0.93242	0.859569	0.797204

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Table 4. Tooth thickness vs. addendum modification coefficient for varying teeth

Addendum modification coefficient	Tooth thickness S			
	Z = 15	Z = 16	Z = 17	Z = 18
-0.4	4.429559	5.252824	6.074167	6.893876
-0.3	4.944174	5.766263	6.586536	7.405266
-0.2	5.459176	6.280038	7.099197	7.916915
-0.1	5.974558	6.794142	7.612146	8.428816
0	6.490311	7.308567	8.125375	8.940964
0.1	7.006426	7.823304	8.638878	9.453353
0.2	7.522891	8.338346	9.152646	9.965975
0.3	8.039697	8.853682	9.666672	10.47883
0.4	8.559831	9.369304	10.18095	10.9919

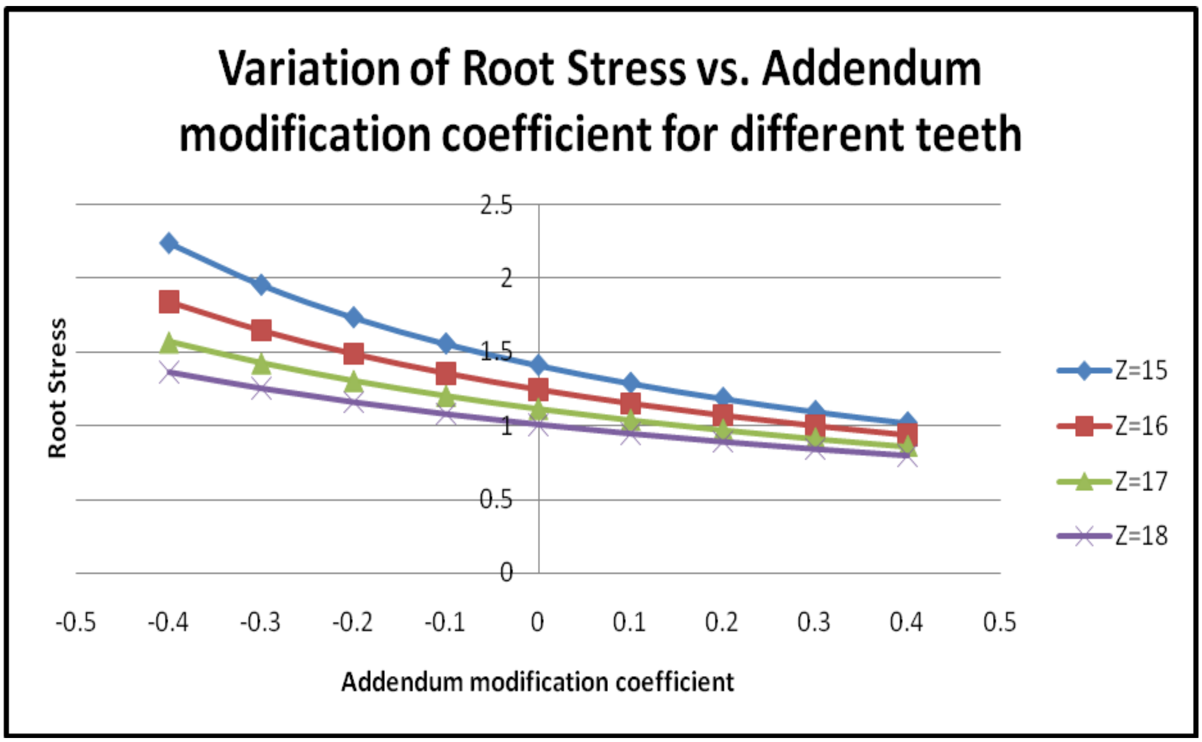

Fig. 9 Root stress vs. addendum modification coefficient for varying teeth

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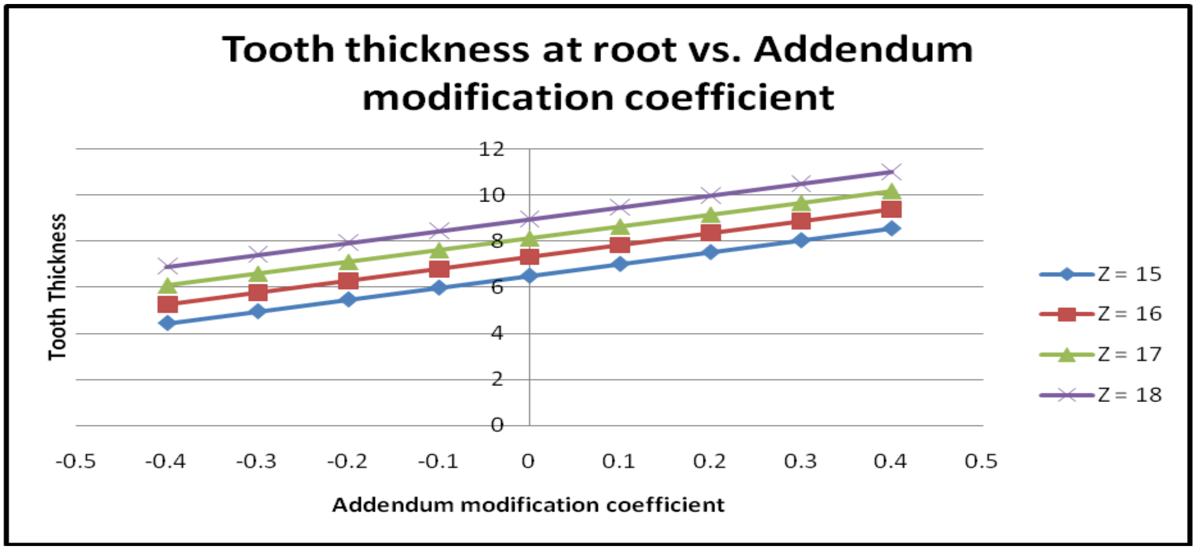

Fig. 10 Tooth thickness vs. addendum modification coefficient for varying teeth

Graphical representation shows that root stress decreases with increase in addendum modification coefficient with increasing teeth. The tooth thickness is also found to increase with increase in addendum modification coefficient and number of teeth.

4.3 Variation in module

The module is varied from 3 to 6 and the variation of root stress and tooth thickness with change in addendum modification coefficient is studied. The tabular and graphical results are shown below:

Table 5. Root stress vs. addendum modification coefficient for varying module

Addendum modification coefficient	Root stress σ_t			
	m = 3	m = 4	m = 5	m = 6
-0.4	2.23724	1.517923	1.151296	0.928579
-0.3	1.95376	1.3439	1.026246	0.830965
-0.2	1.732301	1.204852	0.925193	0.75158
-0.1	1.554628	1.091202	0.841821	0.685731
0	1.408945	0.996548	0.771832	0.630201
0.1	1.287302	0.916458	0.712211	0.582709
0.2	1.184162	0.847771	0.660777	0.541598
0.3	1.095552	0.788172	0.615918	0.505632
0.4	1.018551	0.735927	0.576416	0.473874

Table 6. Tooth thickness vs. addendum modification coefficient for varying module

Addendum modification coefficient	Tooth thickness S			
	m = 3	m = 4	m = 5	m = 6
-0.4	4.429559	6.205214	7.983883	9.764051
-0.3	4.944174	6.883813	.826518	10.77075
-0.2	5.459176	7.562801	9.66544	11.77784
-0.1	5.974558	8.242173	10.51296	12.78531
0	6.490311	8.921918	11.35674	13.79317
0.1	7.006426	9.602027	12.20089	14.80138
0.2	7.522891	10.28249	13.0454	15.80996
0.3	8.039697	10.9633	13.89026	16.81889
0.4	8.556831	11.64444	14.73545	17.82815

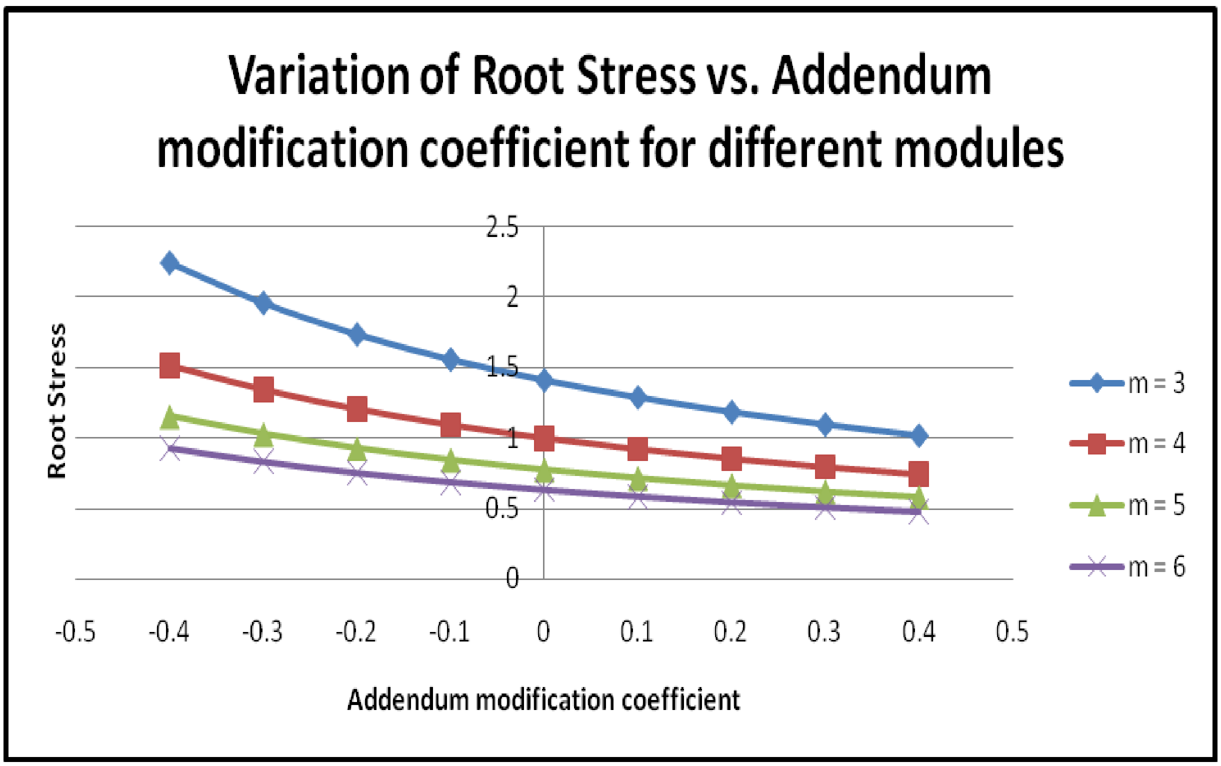

Fig. 11 Root stress vs. addendum modification coefficient for varying modules

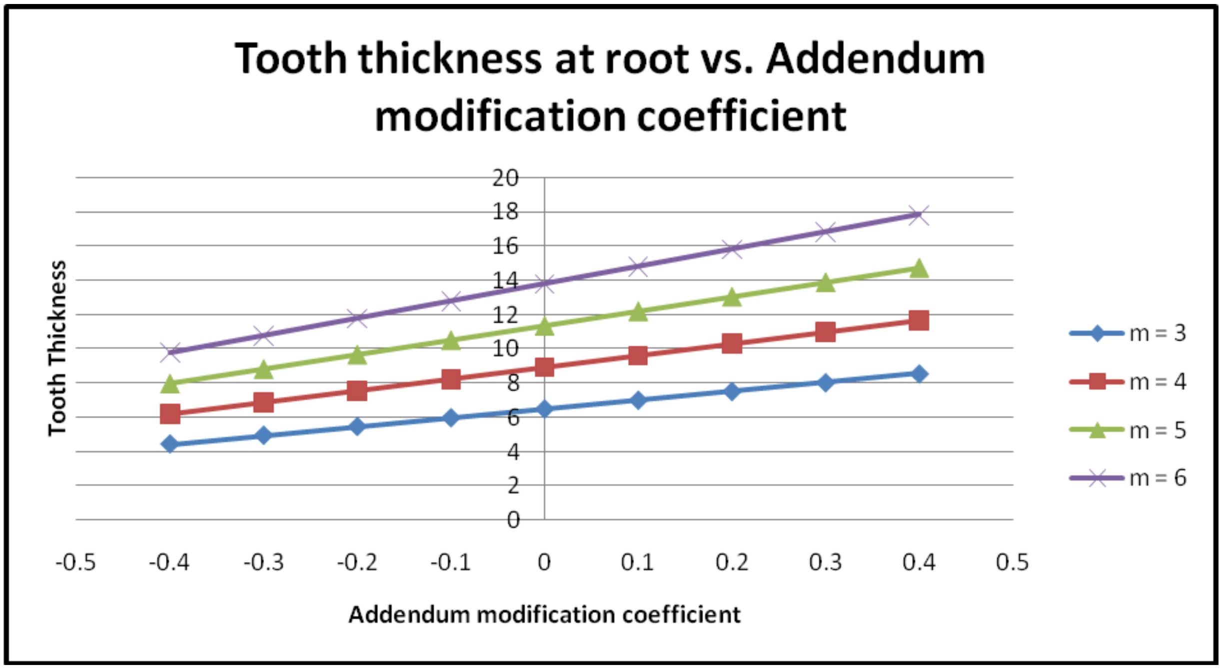

Fig. 12 Tooth thickness vs. addendum modification coefficient for varying modules

Graphical representation shows that root stress decreases with increase in addendum modification coefficient with an increasing module. The tooth thickness also increases with increase in addendum modification coefficient and module.

V. CONCLUSION

Root stress Factor depends upon the Addendum modification coefficient as shown in the graphs. The effect of correction factor on root stress when only the driven gear is corrected is found out for various pressure angles, teeth and modules. Then Root stress factor is also found when both driver gear and the follower gear are modified at the same time in Ansys workbench. The results are discussed in detail in the previous chapters.

The results obtained from the investigation are summarized as follows:

1. The root stress factor for calculating root stress decreases significantly with an increasing addendum modification coefficient. It also decreases with an increase in pressure angle, teeth and module.

2. The tooth thickness at critical section becomes higher with positive addendum modification coefficient. Tooth thickness also increases with an increase in pressure angle, teeth and module. So as the tooth thickness becomes greater at critical section the load carrying capacity of gear increases considerably.

3. The root stress factor decreases further when both the driver and the follower wheel are modified at the same time. Root stress factor attains the lowest value when driver wheel gets positive maximum correction where as the follower gets the negative correction.

4. The values obtained from finite element analysis of the gears also confirm with the theoretical results.

From the above results, it is clear that by doing proper addendum modification, the root stress on gear tooth can be decreased considerably and hence the strength of gear tooth is increased. It is also clear that by suitable addendum modification, the load carrying capacity of the gear can be increased to a great extent. In addition to all these, a smaller number of teeth on gear can be adopted and the most useful advantage is that interference can be avoided. Hence the choice of addendum modification is of great use.

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