APPLICATION OF GA, PSO AND PSO-BFGS FOR THE INVERSE ESTIMATION PROBLEM

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ABSTRACT

This paper deals with the estimation of unknown parameter using evolutionary algorithms such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). A hybrid algorithm is also proposed with the combination of PSO and Broyden Fletcher Goldfarb Shanno Algorithm (BFGS). These algorithms are not only used as optimization techniques but also applied for the estimation of unknown parameters or function appearing in the mathematical model. Initially, the proposed evolutionary algorithms are validated with the available benchmarks in literature. A mathematical model that represents a lumped system in heat transfer is considered to be the forward solution. An inverse problem is proposed to estimate the unknown parameters appearing in the forward model. In order to generate measurement data, the temperatures obtained by solving the forward model for the known parameters are then added with noise at different levels. The unknown parameters appearing in the mathematical model is successfully estimated using these algorithms.

Keywords: Parameter estimation, Genetic Algorithm, Particle Swarm Optimization, BFGS, Evolutionary algorithms.

I. INTRODUCTION

Evolutionary algorithms are most widely used in finding the unknown parameters using optimization concepts. Several popularly used evolutionary algorithms are Genetic Algorithm, Particle Swarm Optimization, Bacterial Foraging Optimization, Ant Colony Optimization, etc.,. The evolution of these new methods is based on analysing the nature and relating their physics with mathematical theories. However, to make use of these algorithms in solving heat transfer problems is quite challenging task as the conventional numerical methods are popularly used to solve such problems. Several work has been attempted in the recent from past to retrieve the unknown parameters using deterministic methods like Steepest descent method, Levenberg method, Conjugate Gradient Method, BFGS method etc.,. A comprehensive discussion about the deterministic and the stochastic methods are available in [1].Many a times, deterministic methods fail to attain global minima/maxima if the initial guess is close to the local minima/maxima. Hence, it is better to use evolutionary algorithm to overcome such problems. Partridge and Wrobel used GA for bio heat transfer problems and proved that GA is an effective tool to findout the locations of skin tumour [2]. Bag et.al [3] used GA in optimizing various process parameters

for variety of weld geometry. Panda and Das estimated the geometrical configurations of a porous fin using GA as an inverse method [4]. The combination of deterministic method and evolutionary algorithms has been used to determine the heat transfer coefficient for continuous casting process using PSO and Gaussian Kernel function [5]. Nusselt number correlation has been developed using PSO for thin fins in narrow cavity based on temperature measurement [6]. The application of PSO has also been reported for radiation problem [7]. Dora and Tortorellit solved the inverse heat conduction problem and compared the results with numerical solutions [8]. A comparison study based on the modified BFGS with PSO resulted in obtaining quality resultsfor solvingstandard benchmark problems [9]. Victoire and Jeyakumar applied PSO-BFGS for different economic dispatch problem [10]. The application of evolutionary algorithm was also found in field of biomechanics where Jaco et al, compared PSO algorithm with GA, BFGS and SQP for identifying the human muscle movements [11]. The work on hybrid PSO algorithm was effectively employed in other engineering applications too [12, 13].

II. PROBLEM STATEMENT FOR A LUMPED SYSTEM

In the present work, the mathematical model represents a lumped system having high thermal conductivity and is subjected to natural convection heat transfer. The initial temperature of the high thermal conductivity material is 100°C and the material is kept in quiescent air at 30°C. Fig. 1 shows the schematic representation of the present model. Let A_s be the surface area in m^2, m is the mass in kg, Cp is the specific heat in J/kgK, τ is the time constant and h is the convectiveheat transfer coefficient in W/m^{2}K. The thermo-physical properties and the experimental temperature distribution are chosen from [14]. Then, the governing equation becomes,

$$mC_p \frac{dT}{dt} = -hA(T - T_\infty) \tag{1}$$

$$\theta = T - T_\infty \tag{2}$$

$$mC_p \frac{dT}{dt} = -hA\theta \tag{3}$$

$$\theta = \theta_i e^{-\frac{t}{\tau}} \tag{4}$$

M=mass
V=volume
P= density
T= initial temperature
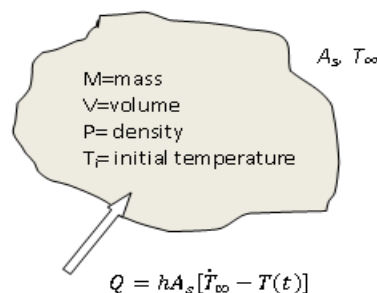
A_s, T_∞

$$Q = hA_s[\dot{T}_\infty - T(t)]$$

Fig.1. Representation of the lumped system.

Table 1. Temperature time history of the system. [14]

S. No	Time,s	Temperature, T
1	10	93.3
2	30	82.2
3	60	68.1
4	90	57.9
5	130	49.2
6	180	41.4
7	250	36.3
8	300	32.9

The main aim of the present work is to estimate the value of the time constant (τ) by using the temperature time history of the system as shown in Table 1. In order to estimate the time constant, an objective function is defined based on the least squares as shown in equation (5).

$$S(\tau) = \sum_{p=1}^{N}[\theta_p - \theta_p(\tau)]^2 \tag{5}$$

Where τ is the unknown parameter. θ_p is the pth observation from the measurements, $\theta_p(\tau)$ is the temperature values obtained by solving the equation(4).

III.GENETIC ALGORITHM

The Genetic algorithm was developed by Goldberg and Holland [15]. The basic steps in the genetic algorithm include selection of the input parameters called population initialization, evaluation of fitness function, reproduction, cross over, mutation and final solution. In the Initialization step, a range of input values is defined in terms of corresponding binary values. These values are called chromosomes. A range of input is referred to as set of chromosomes.

Example: For a value of 1500, corresponding binary number is

Guess number: 1500

Binary: 10111011100

Each individual (0 or 1) is referred to as chromosomewhich contains certain number of digits called genes. In the above example, the number of genes is 11. Fitness function is evaluated for each of these input values. The values are then sorted outbased on the maximum, mean and minimum of the fitness function values.

IV.PSO ALGORITHM

In 1995, Kennedy and Eberhart developed Particle Swarm Optimization (PSO) as a robust technique which can be implemented to various inverse problems [16]. In comparison with the existing biological algorithms, PSO uses lesser parameters to solve the objective function. In a search space of N dimension with M particles moving in a swarm, the position and the velocity vectors of the ith particle are given by $X_i=(x_{i1},x_{i2},....,x_{iM})$ and $V_i=(v_{i1},v_{i2},.....,v_{iN})$. In every generation, the addition of displacement to the present location X_i^n will give a new position X_i^{n+1} which is given by,

$$X_i^{n+1} = X_i^n + V_i^{n+1} \tag{6}$$

The corresponding updated velocity V_i^{n+1} from the previous velocity V_i^n of the each particle in the direction of $P_{best.i}$ and $g_{best.i}$ is given as,

$$V_i^{n+1} = W_a V_i^n + c_1.r_1.(P_{best.i} - X_i^n) + c_2.r_2.(g_{best.i} - X_i^n) \tag{7}$$

Particles contain particle best according to fitness value that contains the information of their previous position. $P_{best.i} = (p_{i1}, p_{i2}, ... p_{iN})$. The best position of the swarm particle isidentified as global best$g_{best.i} = (p_{g1}, p_{g2}, ... p_{gN})$. The second term in Equation (7) is called as cognition part associated with cognition learning coefficient c_1,which can control the movement of the particle. The third term is the social component associated with c_2, that makes the particles to decided next position. r_1 and r_2 are the random numbers whose value lie from 0 to 1. The sum of the values of c_1 and c_2 should be lesser or equal to 4. Variable w is the inertia weighting function which has an important role in controlling exploration capacities of the swarms. The value of w lies between 0 and 1. For the present work, variable W_a value is expressed as

$$W_a = \frac{(maximum\ iteration - current\ iteration)}{maximum\ iteration} \tag{8}$$

V.PSO-BFGS ALGORITHM

PSO sometimes suffer premature convergence [17]. This can be overcome by combining PSO with BFGS method. The method improves the particles to search locally with a better solution. For nonlinear optimization, the use of BFGS method is very efficient as its performance is found to be more accurate compared to other optimization algorithms. The procedure for BFGS is explained as below [18].

Step 1: Set the value of initial point x^0 and convergence criterion ε>0.

Step 2: Initialize a Hessian matrix$H^0 = I_n$, calculate the gradient $d^0 = \nabla J(x^0)$

Hessian matrix H^{k+1} of the Lagrangian function is evaluated in each iteration replacing the previous Hessian matrixH^k. The search direction y^k is given by

Step 3:

$$y^k = -H^k d^k \tag{9}$$

Step 4: Set the values

$$d^{k+1} = \nabla J(x^{k+1}) \tag{10}$$

$$x^{k+1} = x^k + \alpha_k y^k \tag{11}$$

The value α_k can be computed by the below equation (12),

$$J(x^k + \alpha_k y^k) = \min J(x^k + \alpha y^k) \ for \ \alpha \geq 0 \tag{12}$$

Step 5: If $\|d^{k+1}\| \leq \varepsilon$, calculation is stopped and the output is the solution. Else go to

Step 6: set $g^k = d^{k+1} - d^k$ and $\beta^K = x^{k+1} - x^k$ evaluate H^{k+1} which is given by

$$H^{k+1} = H^k + \left(1 + \frac{g^k H^k g^{kT}}{\beta^{kT} g^k}\right)\frac{\beta^k \beta^{kT}}{\beta^{kT} g^k} - \frac{\beta^k g^{kT} H^k + H^k g^k \beta^{kT}}{\beta^{kT} g^k} \tag{13}$$

Step 7: Set the value k to next iteration k+1 and go to step 3.

For PSO-BFGS method, after finding the global best for the current iteration using PSO [10]

Step 8: if $g_{best}^t < g_{best}^{t-1}$, solve the inverse problem with BFGS algorithm using the present global best g_{best}^t.

Step 9: Replace the g_{best}^t value with the final solution obtained using BFGS method, else

Step 10: go to step 4.

Step 11: change the velocities and positions.

Step 12: iteration continued till the stopping criterion is reached.

VI.ESTIMATION OF THE UNKNOWN PARAMETER

The unknown parameter appearing in the mathematical model is now solved using GA, PSO and PSO-BFGS as inverse methods. Before attempting the solution to the present inverse problem, a benchmark problem is solved using the above mentioned evolutionary algorithms to provide a firm establishment of the ability of these algorithms. The bench mark problem used is Rosenbrock banana function [19] which is given by Equation (14)

$$f(x) = 100(y - x^2)^2 + (1 - x)^2 \tag{14}$$

The range of x, y is chosen between -5 and 10. The retrieved values using the mentioned algorithms are tabulated in Table 2. The minimum of the banana function lies at (1,1). The 3D plot of the Banana function is shown in Fig.2. Fig.3 shows the evaluation of Rosenbrock function using GA, PSO and PSO-BFGS and it is evident that the in-house code was able to predict the values of x and y accurately.
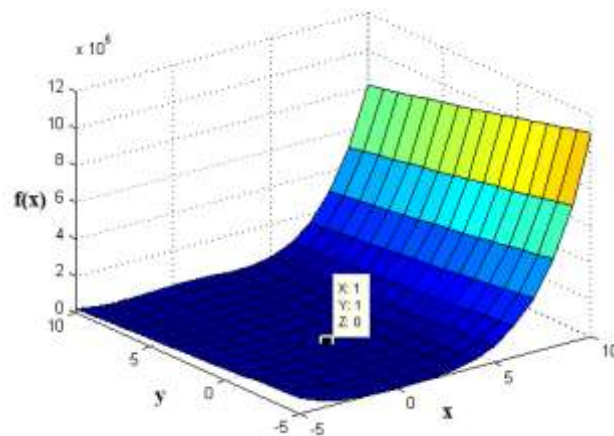

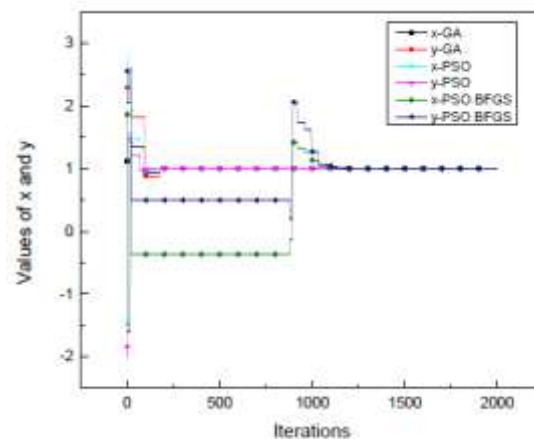**Fig.2.** 3D representation of Banana function.

Fig.3. Minimum values of Rosenbrock banana function.

Table 2. Retrieved values of x and y using mentioned algorithms.

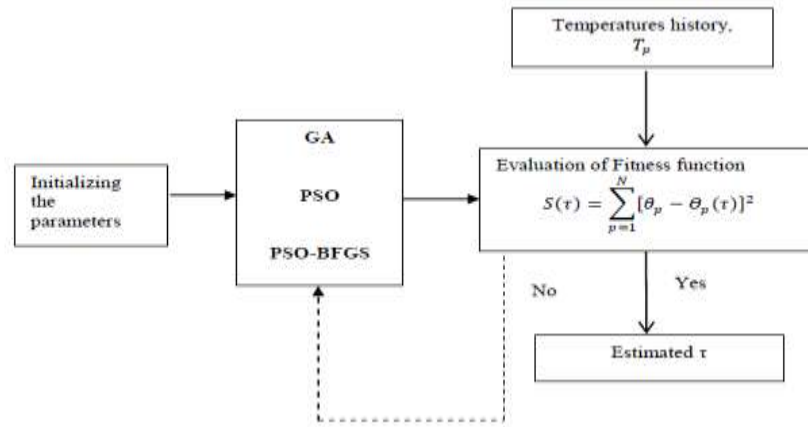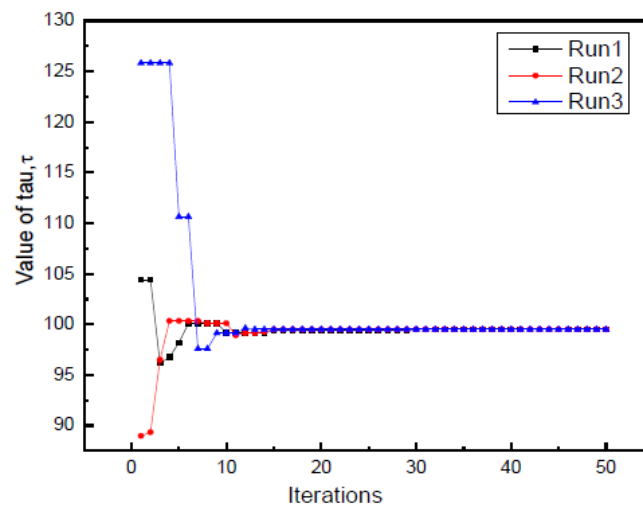|	GA			PSO			PSO-BFGS		
Runs	x	y	Time,s	x	y	Time,s	X	y	Time,s
1	1	1	0.491	1	1	0.27	1	1	0.284
2	1	1	0.484	1	1	0.298	1	1	0.277
3	1	1	0.512	1	1	0.287	1	1	0.292

Fig.4. Overall representation of the work.

Fig.5. Retrieval of time constant for different runs using GA.

Figure 4 represents the overall representation of the present approach and Fig.5 shows the retrieval of time constant (τ) using GA method and the solution is obtained for several trials and three such runs are reported in Table 3 for the same range of inputs. The number of iterations is set to 50. The value of τ is randomly chosen within the range [10 500]. The mutation rate, chromosomes number and gene number are assigned to be 0.1, 10 and 12, respectively. Each chromosome in the range assumed represents the values of τ. The difference between Temperatures θ_p and actual temperature T_i is calculated in the objective function. The minimum, mean and maximum values of the fitness function as shown in Fig. 6 correspond to best, average and worst chromosomes

which will be considered for the future iterations. The effect of algorithm parameters like chromosome number and gene number is also studied and the values are tabulated in Table 4 and 5 respectively.

Table 3. Estimated values for the τ using GA for several runs.

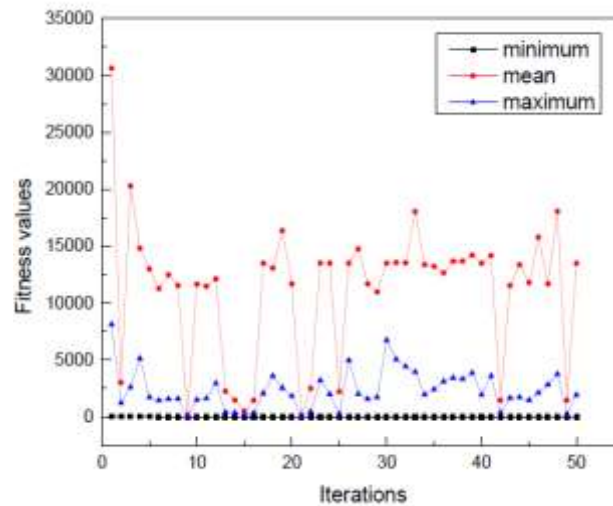| Runs	τ	Time,s
1	99.55	0.234
2	99.49	0.244
3	99.55	0.246

Fig.6. Fitness values of the GA.

Table 4. Effect of chromosome number on estimation using GA.

Chromosome number	τ	Time,s
10	99.5	0.256
20	101.89	0.27
30	99.55	0.31

Table 5. Effect of gene number on estimation using GA.

genes	τ	Time, s
8	102.23	0.212
12	99.49	0.244

The estimation of τ is further carried out using PSO algorithm for the same range of τ value. To evaluate the objective function, swarms, number of particles, are also initialized. In the present study, the particles represent a range of τ. For every iteration, the particle's previous best and the global best are updated. The values of c_1 and c_2 are set to 1.43 and 1.43, respectively. After the evaluation of fitness function, the personal best and global best values are updated appropriately. In the next iteration, the updated velocities and positions of each particle are considered.The estimated values of three such runs are tabulated in Fig.7 and the values are tabulated in Table 6. The position and velocities are generated randomly. Similar to GA, effect of number of particles on estimation is tabulated in Table 7.
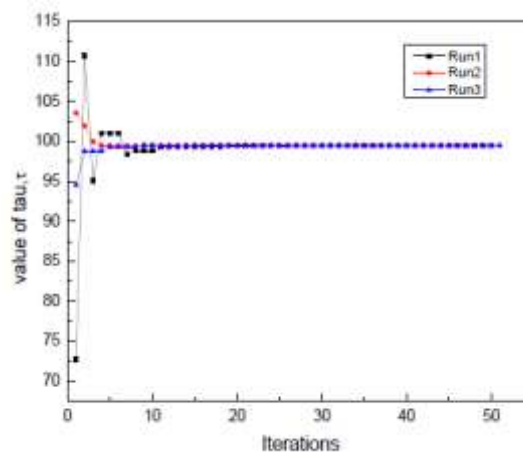

Fig.7. Retrieval of tau for different runs using PSO.

Table 6. Estimation of τ using PSO.

Runs	τ	Time, s
1	99.46	0.21
2	99.46	0.176
3	99.46	0.2

Table 7. Effect of particle numbers on estimation using PSO.

Particle numbers	τ	Time, s
10	99.46	0.21
20	99.46	0.18
30	99.46	0.21

The adaptation of hybrid method, PSO-BFGS algorithm is carried out using the same initial guess as PSO but the algorithm makes use of hessian matrix to improve the solution. The value of α and direction is calculated to

find the increment for new modified value of global best of the particle. Results of PSO-BFGS method are shown in Table 8for three different runs and plotted in Fig.8.

Table 8.Estimation of τ using PSO BFGS.

Runs	τ	Time, s
1	99.46	0.4
2	99.46	0.32
3	99.46	0.28

The computational time required for the estimation of heat flux using PSO-BFGS algorithm was observed to be slightlymore compared to other two algorithms because it needs to find out appropriate value of α to predict the increment of the next parameter. The effect of number of particles is also mentioned for the proposed hybrid scheme which is shown in Table 9.

Table 9. The effect of number of particles on estimation using PSO-BFGS.

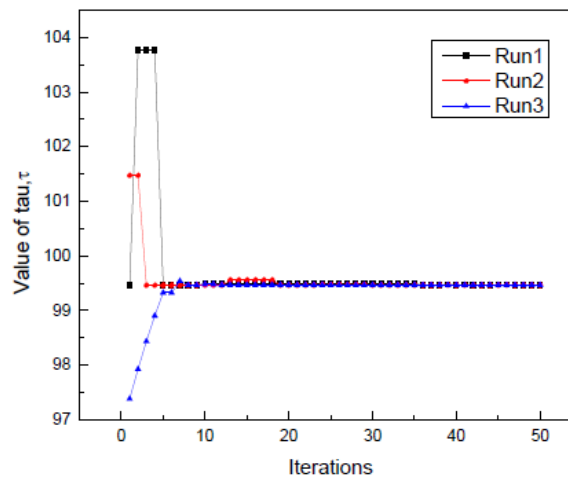| Particle numbers	τ	Time, s
10	99.46	0.28
20	99.46	0.3
30	99.46	0.35

Fig.8. Retrieval of tau for different runs using PSO-BFGS.
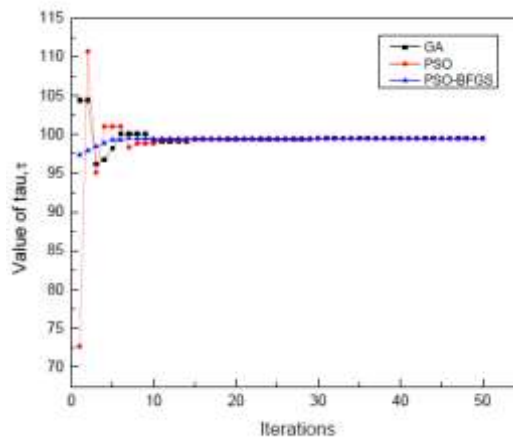
Fig.9. Comparison of estimated values of τ for different algorithms.

Figure 9 shows the comparison for the convergence of estimated τ values for mentioned algorithms. Fig.10 shows the comparison of fitness values for GA, PSO and PSO BFGS algorithms respectively. From the above results on the estimation of time constant (τ), it can be observed that the all the three algorithms were successful in estimating τ. The study on effect of algorithms parameters like number of population, gene number for GA showed some inconsistency in estimating the value of τ as shown in the table 4 and 5 respectively.
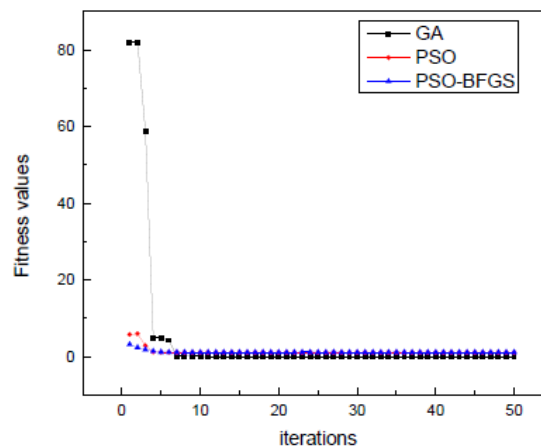

Fig.10. Comparison of fitness function values for different algorithms.

In case of PSO and PSO-BFGS, the estimation was consistent for all the runs. It depends on the complexity of the problem. Generally, for any evolutionary algorithm, the increase in the population size or particle size will increase the computational cost and there will be faster convergence of the solution as it provides a large search space for the estimation process [17, 20].The results obtained by all the mentioned algorithms were comparable and with the promising results obtained from PSO-BFGS one can apply this hybrid technique for complex problems in any application.

VII.CONCLUSION

In this paper study, the solution of a lumped system in heat transfer is solved using evolutionary algorithms GA, PSO and PSO-BFGS. The in-house code has been verified with the benchmark problems. The inverse estimation is accomplished for the measurement data available in literature. In order to solve the time constant value, the objective function is framed according to least squares method. Thus, results obtained with all the algorithms show good estimates in the field of parameter estimation. Later, itwas concluded that the PSO-BFGS,a new hybrid optimization algorithm, can effectively be used to solve the heat transfer problems. All the three algorithms were found effective in parameter estimation hence provides a great hope in estimating the unknown parameters in the field of heat transfer.

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