Common Fixed Point Theorem of Compatible Type of (K) Satisfying Integral type Inequality on Intuitionistic Fuzzy Metric Spaces

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ABSTRACT

In this paper we prove common fixed point theorem for compatible of type (K) satisfying integral type inequality in intuitionistic fuzzy metric space and obtain a common fixed point theorem for self mapping in intuitionistic fuzzy metric space satisfying integral type inequality.

Keywords: Intiutionistic Fuzzy Metric Space, Fixed Point, Compatible Maps Of Type (K).

I INTRODUCTION

An essential feature of metric space is that for any two points in the metric space, there is defined a positive number called the distance between the points. Zadeh [20] introduced the concept of fuzzy set A in X is a function with domain X and value in [0, 1]. Deng [3], Erceg [4], Fang [5], George and Veeramani [7], Kaleva and Seikkala[10], Kramosil and Michalek [11] have introduced the concept of fuzzy metric spaces in different ways. Atanassove [1] introduced and studied the concept of intuitionistic fuzzy sets, In 2004, Park [15] defined the notion of Intuitionistic fuzzy metric space with the help of continuous *t*-norms and continuous *t*-conorms as generalization of fuzzy metric space due to George and Veeramani[7]. Several authors [12, 13, 14, 16] proved some fixed points theorem in intuitioistic fuzzy metric spaces. Further Coker [2] introduced the concept of Intuitionistic fuzzy topological spaces. Turkoglu et al. [17] gave a generalization of Jungck's common fixed point theorem [9] to Intuitionistic fuzzy metric spaces. In this paper we prove a common fixed point theorem in Intuitionistic fuzzy metric space for compatible mapping type of (K) satisfying integral type inequality.

II. PRELIMINARIES

Definition 2.1:

A binary operation $*$: $[0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ is satisfies the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2:

A binary operation $\Diamond$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous *t*-conorm if $\Diamond$ is satisfies the following conditions:

(i) $\Diamond$ is commutative and associative;

(ii) $\Diamond$ is continuous;

(iii) $a \Diamond 0 = a$ for all $a \in [0, 1]$;

(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3:

A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$, $s, t > 0$,

(i) $M(x, y, t) + N(x, y, t) \leq 1$

(ii) $M(x, y, 0) = 0$

(iii) $M(x, y, t) = 1$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t) \neq 0$ for $t \neq 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

(vi) $M(x, y, \cdot): [0,\infty) \rightarrow [0, 1]$ is continuous.

(vii) $\lim_{t\to\infty} M(x, y, t) = 1$

(viii) $N(x, y, 0) = 1$

(ix) $N(x, y, t) = 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t) \neq 0$ for $t \neq 0$;

(xi) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$;

(xii) $N(x, y, \cdot): [0,\infty) \rightarrow [0, 1]$ is continuous.

(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 2.4:

Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ is said to be

(i) Convergent to a point x∈X if

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \text{ and } \lim_{n\to\infty} N(x_n, x, t) = 0$$

For all t > 0

(ii) Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$$

For all t > 0 and p > 0

Definition 2.5:

A sequence $\{x_n\}$ in an intuitioistic fuzzy metric space (X, M, N, $*$, $\diamond$) is called complete if and only if every Cauchy sequence in X is convergent.

Definition 2.6:

Let S and T be self mapping of an intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$). Then a pair (S, T) is said to be compatible if

$$\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ST x_n, TSx_n, t) = 0$$

For all t > 0, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some u∈X.

Definition 2.7:

Let S and T be self mapping of an intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$). Then a pair (S, T) is said to be compatible of type (A) if

$$\lim_{n\to\infty} M(STx_n, TTx_n, t) = 1 \text{ and } \lim_{n\to\infty} M(TS x_n, SSx_n, t) = 1$$

and

$$\lim_{n\to\infty} N(STx_n, TTx_n, t) = 0 \text{ and } \lim_{n\to\infty} N(TS x_n, SSx_n, t) = 0$$

For all t > 0, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some u∈X.

Definition 2.8:

Let S and T be self mapping of an intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$). Then a pair (S, T) is said to be compatible of type (P) if

$\lim_{n\to\infty} M(TTx_n, SSx_n, t) = 1$ and $\lim_{n\to\infty} N(TTx_n, SSx_n, t) = 0$

For all t > 0, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some u∈X.

Definition 2.9:

Let S and T be self mapping of an intuitionistic fuzzy metric space (X, M, $N, *, \diamond$) are called reciprocally continuous on X if $\lim_{n\to\infty} STx_n = Sx$ and $\lim_{n\to\infty} TSx_n = Tx$ whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some u∈X.

Definition 2.10:

Let S and T be self mapping of an intuitionistic fuzzy metric space (X, M, $N, *, \diamond$).. Then a pair (S, T) is said to be compatible of type (K) iff $\lim_{n\to\infty} M(SSx_n, TTx, t) = M(Tx, Sx, t)$

$\lim_{n\to\infty} SSx_n = Tx$ and $\lim_{n\to\infty} TTx_n = Sx$, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some u∈X.

Example

Let X = [0, 10] with the usual metric d (x, y) = | x - y |, define M(x, y, t) = $\frac{t^2}{t^2 + d(x,y)}$ for all x, y ∈X, t > 0 and $a * b = \min(a, b)$ for all a, b ∈[0, 1] then (X, M, $*$) is a fuzzy metric space . We define self-mappings S and T as Sx =10, Tx = 0 for $x \in [0, 5] - \{\frac{5}{2}\}$, S$x$ = 0, Tx = 10 for $x = \frac{5}{2}$ and S$x = \frac{10-x}{2}$, T$x = \frac{x}{2}$ for x∈(5, 10].

Then, S and T are not continuous at $x = 5, \frac{5}{2}$.

Consider a sequence $\{x_n\}$ in X such that $x_n = 5 - \frac{1}{n}$ for all n ∈ N.

Then we have Sx_n = $\frac{(10-x_n)}{2} \to \frac{5}{2} = x$ and Tx_n= $\frac{x_n}{2} \to \frac{5}{2} = x$

Also, we have SSx_n = S$\frac{(5-x_n)}{2}$ = $10 \to 10$, STx_n = $S\frac{(x_n)}{2}$ = $10 \to 10$, T(x) =10 and TTx_n = T$\frac{(x_n)}{2}$ = $0 \to 0$, TSx_n = T$\frac{(5-x_n)}{2}$ = $0 \to 0$, S(x) =0. Therefore, (S, T) is compatible of type (K) but the pair (S, T) is neither compatible nor compatible of type (A) (compatible type of (p), reciprocal continuous).

Lemma: 2.1

Let (X, M, $N, *, \diamond$) be an intuitionistic fuzzy metric space. Then for all x, y in X, M$(x, y, .)$ is non decreasing.

Lemma: 2.2

Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space. If there exist q$\in (0,1)$ such that M$(x, y, qt) \geq$ M(x, y, t) and N$(x, y, qt) \leq$ N(x, y, t) for all x, y and t > 0 then $x = y$.

Lemma: 2.3

The only t-norm $*$ satisfying $r * r \geq r$ for all $r \in [0, 1]$ is the minimum t-norm, that is $a * b$ = min {a, b} for all a, b [0, 1].

Main Theorem

Theorem:

Let (X, M, N, $*$, $\diamond$) be a complete Intuitionistic fuzzy metric space and A, B, S and T be self mapping of X satisfying the following conditions:

(i) A(X) ⊆ T(X) and B(X) ⊆ S(X)

(ii) $\int_0^{M(Ax,By,kt)} \psi(t)dt \geq \int_0^{M(Ax,Sx,t)*M(By,Ty,t)*M(Sx,Ty,t)*M(Ax,Ty,\propto t)*M(Bx,Tx,(2-\propto)t)} \psi(t)dt$

$\int_0^{N(Ax,By,kt)} \psi(t)dt \leq \int_0^{N(Ax,Sx,t)\diamond N(By,Ty,t)\diamond N(Sx,Ty,t)\diamond N(Ax,Ty,\propto t)\diamond N(Bx,Tx,(2-\propto)t)} \psi(t)dt$

Where $\propto$: [0, 1] → [0, 1] is continuous function such that $r(t) > t$ for some 0$< t < 1$ for all x, y $\in$ X , $k \in (0,1)$, $\propto \in (0,2)$ and

(iii) S and T are continuous.

If (A, S) and (B, T) compatible of type of (K), then A, B, S and T have a unique common fixed point.

Proof:

(i) A(X) ⊆ T (X) and B(X) ⊆ S(X),

So for any $x_0 \in$ X, there exists $x_1 \in$ X such that $Ax_0 = Tx_1$ and for this x_1, there exists $x_2 \in$ X such that $Bx_1 = Sx_2$. Inductively, we define a sequence $\{y_n\}$ in X such that

y_{2n-1} = A$x_{2n-2} = Tx_{2n-1}$ and $y_{2n} = Bx_{2n-1} = Sx_{2n}$ for $all\ n$= 0, 1, 2…

Taking $x = x_{2n}$ and $y = x_{2n+1}$ in (ii), we get

$$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \psi(t)dt = \int_0^{M(Ax_{2n},Bx_{2n+1},kt)} \psi(t)dt$$

$$\geq \int_0^{\substack{M(Ax_{2n},Sx_{2n},t)*M(Bx_{2n+1},Tx_{2n+1},t)*M(Sx_{2n},Tx_{2n+1},t)* \\ M(Ax_{2n},Tx_{2n+1},\propto t)*M(Bx_{2n},Tx_{2n},(2-\propto)t)}} \psi(t)dt$$

Now we put $\propto = 1-q$ with $q \in (0,1)$, we get

$$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \psi(t)dt \geq \int_0^{\substack{M(Ax_{2n},Sx_{2n},t)*M(Bx_{2n+1},Tx_{2n+1},t)*M(Sx_{2n},Tx_{2n+1},t)* \\ M(Ax_{2n},Tx_{2n+1},(1-q)t)*M(Bx_{2n},Tx_{2n},(2-(1-q))t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{M(y_{2n+1},y_{2n},t)*M(y_{2n+2},Tx_{2n+1},t)*M(y_{2n},y_{2n+1},t)* \\ M(y_{2n+1},y_{2n+2},t)*M(y_{2n+1},y_{2n},(1+q))t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{M(y_{2n},y_{2n+1},t)*M(y_{2n+1},y_{2n+2},t)*M(y_{2n},y_{2n+1},t)* \\ M(y_{2n+1},y_{2n+2},t)*M(y_{2n},y_{2n+1},t)*M(y_{2n},y_{2n+1},q\,t)}} \psi(t)dt$$

$$\geq \int_0^{M(y_{2n},y_{2n+1},t)*M(y_{2n+1},y_{2n+2},t)*M(y_{2n},y_{2n+1},qt)} \psi(t)dt$$

$$\geq \int_0^{M(y_{2n},y_{2n+1},t)*M(y_{2n+1},y_{2n+2},t)} \psi(t)dt$$

And

$$\int_0^{N(y_{2n+1},y_{2n+2},kt)} \psi(t)dt = \int_0^{N(Ax_{2n},Bx_{2n+1},kt)} \psi(t)dt$$

$$\leq \int_0^{\substack{N(Ax_{2n},Sx_{2n},t)\diamond N(Bx_{2n+1},Tx_{2n+1},t)\diamond N(Sx_{2n},Tx_{2n+1},t)\diamond \\ N(Ax_{2n},Tx_{2n+1},\propto t)\diamond N(Bx_{2n},Tx_{2n},(2-\propto)t)}} \psi(t)dt$$

Now we put $\propto = 1-q$ with $q \in (0,1)$, we get

$$\leq \int_0^{\substack{N(Ax_{2n},Sx_{2n},t)\diamond N(Bx_{2n+1},Tx_{2n+1},t)\diamond N(Sx_{2n},Tx_{2n+1},t)\diamond \\ N(Ax_{2n},Tx_{2n+1},(1-q)t)\diamond N(Bx_{2n},Tx_{2n},(2-(1-q))t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{N(y_{2n+1},y_{2n},t)\diamond N(y_{2n+2},Tx_{2n+1},t)\diamond N(y_{2n},y_{2n+1},t)\diamond \\ N(y_{2n+1},y_{2n+2},t)\diamond N(y_{2n+1},y_{2n},(1+q))t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{N(y_{2n},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n+2},t)\diamond N(y_{2n},y_{2n+1},t)\diamond \\ N(y_{2n+1},y_{2n+2},t)\diamond N(y_{2n},y_{2n+1},t)\diamond N(y_{2n},y_{2n+1},q\,t)}} \psi(t)dt$$

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$$\leq \int_0^{N(y_{2n},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n+2},t)\diamond N(y_{2n},y_{2n+1},q\,t)} \psi(t)dt$$

$$\leq \int_0^{N(y_{2n},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n+2},t)} \psi(t)dt$$

From lemma 2.1 and 2.3 we have

$$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \psi(t)dt \geq \int_0^{M(y_{2n},y_{2n+1},t)} \psi(t)dt$$

and $\int_0^{N(y_{2n+1},y_{2n+2},kt)} \psi(t)dt \leq \int_0^{N(y_{2n},y_{2n+1},t)} \psi(t)dt$

(1)

Similarly, we have

$\int_0^{M(y_{2n+2},y_{2n+3},kt)} \psi(t)dt \geq \int_0^{M(y_{2n+1},y_{2n+2},t)} \psi(t)dt$ and

$$\int_0^{N(y_{2n+2},y_{2n+3},kt)} \psi(t)dt \leq \int_0^{N(y_{2n+1},y_{2n+2},t)} \psi(t)dt$$

(2)

From (1) and (2), we have

$$\int_0^{M(y_{n+1},y_{n+2},kt)} \psi(t)dt \geq \int_0^{M(y_n,y_{n+1},t)} \psi(t)dt$$

And

$$\int_0^{N(y_{n+1},y_{n+2},kt)} \psi(t)dt \leq \int_0^{N(y_n,y_{n+1},t)} \psi(t)dt$$

(3)

From (3), we have $\int_0^{M(y_{n+1},y_{n+2},kt)} \psi(t)dt \geq \int_0^{M\left(y_n,y_{n+1},\frac{t}{k}\right)} \psi(t)dt \geq \int_0^{M\left(y_{n-1},y_{n+1},\frac{t}{k^2}\right)} \psi(t)dt \geq \ldots \geq$

$$\int_0^{M\left(y_1,y_2,\frac{t}{k^n}\right)} \psi(t)dt \to 1 \text{ as } n \to \infty$$

and

$$\int_0^{N(y_{n+1},y_{n+2},kt)} \psi(t)dt \leq \int_0^{N(y_n,y_{n+1},t)} \psi(t)dt \int_0^{N\left(y_{n-1},y_{n+1},\frac{t}{k^2}\right)} \psi(t)dt \leq \ldots \leq$$

$$\int_0^{N\left(y_1,y_2,\frac{t}{k^n}\right)} \psi(t)dt \to 1 \text{ as } n \to \infty$$

So $\int_0^{M(y_n,y_{n+1},t)} \psi(t)dt \to 1$ as $n \to \infty$ for any t > 0.For each $\varepsilon > 0$ and each t > 0, we can choose $n_0 \in N$ such that $\int_0^{M(y_n,y_{n+1},t)} \psi(t)dt > 1-\varepsilon$ for all $n > n_0$. For $n, m \in N$, we suppose $m \geq n$. Then we have that

$$\int_0^{M(y_n,y_m,t)} \psi(t)dt \geq \int_0^{M\left(y_n,y_{n+1},\frac{t}{m-n}\right)} \psi(t)dt * \int_0^{M\left(y_{n+1},y_{n+2},\frac{t}{m-n}\right)} \psi(t)dt * \ldots$$

$$* \int_0^{M\left(y_{n+1},y_{n+2},\frac{t}{m-n}\right)} \psi(t)\, dt \geq 1-\varepsilon * 1-\varepsilon * \ldots \quad (m-n) \text{ times. This implies}$$

$\int_0^{M(y_n,y_m,t)} \psi(t)dt \leq (1-\varepsilon)$ and hence $\{y_n\}$ is a Cauchy sequence in X.

Since (X, M, N, $*$, $\diamond$) is complete, $\{y_n\}$ converges to some point $z \in X$, and so that $\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$, $Tx_{2n-1}\}$ also converges to z. Since (A, S) and (B, T) are compatible of type (K), we have

$$AAx_{2n-2} \to Sz,\ SSx_{2n} \to Az, BBx_{2n-1} \to Tz, TTx_{2n-1} Bz \qquad (4)$$

From (ii), we get

$$\int_0^{M(AAx_{2n-2},BBx_{2n-1},kt)} \psi(t)dt$$

$$\geq \int_0^{\substack{(M(AAx_{2n-2},SAx_{2n-2},t) * M(BBx_{2n-1},TBx_{2n-1},t) * M(SAx_{2n-2},TBx_{2n-1},t) * \\ M(AAx_{2n-2},TBx_{2n-1},\propto t) * M(BAx_{2n-2},TAx_{2n-2},(2-\propto)t)}} \psi(t)dt$$

Again taking limit as $n \to \infty$ and using (4), we have

$$\int_0^{M(Sz,Tz,kt)} \psi(t)dt \geq \int_0^{\substack{M(Sz,Sz,t) * M(Tz,Tz,t) * M(Sz,Tz,t) * \\ M(Sz,Tz,(1-q)t) * M(Bz,Tz,(2-(1-q)t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{M(Sz,Sz,t) * M(Tz,Tz,t) * M(Sz,Tz,t) * \\ M(Sz,Tz,t) * M(Bz,Tz,t) * M(Bz,Tz,qt)}} \psi(t)dt$$

$$\geq \int_0^{\substack{1 * 1 * \\ M(Sz,Tz,t) * M(Bz,Tz,t)}} \psi(t)dt$$

$$\geq \int_0^{M(Sz,Tz,t) * M(Bz,Tz,t)} \psi(t)dt$$

$$\geq \int_0^{M(Sz,Tz,t) * 1} \psi(t)dt$$

$$\geq \int_0^{M(Sz,Tz,t)} \psi(t)dt$$

and

$$\int_0^{N(AAx_{2n-2},BBx_{2n-1},kt)} \psi(t)dt$$
$$\leq \int_0^{\substack{(N(AAx_{2n-2},SAx_{2n-2},t) \diamond N(BBx_{2n-1},TBx_{2n-1},t) \diamond N(SAx_{2n-2},TBx_{2n-1},t) \diamond \\ N(AAx_{2n-2},TBx_{2n-1},\alpha t) \diamond N(BAx_{2n-2},TAx_{2n-2},(2-\alpha)t)}} \psi(t)dt$$

Again taking limit as $n \to \infty$ and using (4), we have

$$\int_0^{N(Sz,Tz,kt)} \psi(t)dt \leq \int_0^{\substack{N(Sz,Sz,t) \diamond N(Tz,Tz,t) * N(Sz,Tz,t) \diamond \\ N(Sz,Tz,(1-q)t) \diamond N(Bz,Tz,(2-(1-q)t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{N(Sz,Sz,t) \diamond N(Tz,Tz,t) \diamond N(Sz,Tz,t) \diamond \\ N(Sz,Tz,t) \diamond N(Bz,Tz,t) \diamond N(Bz,Tz,qt)}} \psi(t)dt$$

$$\leq \int_0^{\substack{0 \diamond 0 \diamond \\ N(Sz,Tz,t) \diamond N(Bz,Tz,t)}} \psi(t)dt$$

$$\leq \int_0^{N(Sz,Tz,t) \diamond N(Bz,Tz,t)} \psi(t)dt$$

$$\leq \int_0^{N(Sz,Tz,t) \diamond 0} \psi(t)dt$$

$$\leq \int_0^{N(Sz,Tz,t)} \psi(t)dt$$

Sz = Tz

From (ii), we get

$$\int_0^{M(Az,BBx_{2n-1},kt)} \psi(t)dt \geq \int_0^{\substack{(M(Az,Sz,t) * M(BBx_{2n-1},TBx_{2n-1},t) * M(Sz,TBx_{2n-1},t) * \\ M(Az,TBx_{2n-1},\alpha t) * M(Bz,Tz,(2-\alpha)t)}} \psi(t)dt$$

Again taking limit as $n \to \infty$ and using (4), we have

$$\int_0^{M(Az,Tz,kt)} \psi(t)dt \geq \int_0^{\substack{M(Az,Sz,t) * M(Tz,Tz,t) * M(Sz,Tz,t) * \\ M(Az,Tz,(1-q)t) * M(Bz,Tz,(2-(1-q)t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{1 * M(Sz,Sz,t) * \\ M(Az,Tz,t) * M(Bz,Tz,t) * M(Bz,Tz,qt)}} \psi(t)dt$$

$$\geq \int_0^{\substack{1 * 1 * 1 \\ M(Az,Tz,t) * M(Bz,Tz,t)}} \psi(t)dt$$

$$\geq \int_0^{M(Az,Tz,t)*1} \psi(t)dt$$

$$\geq \int_0^{M(Az,Tz,t)} \psi(t)dt$$

$Az = Tz$

and

$$\int_0^{N(Az,BBx_{2n-1},kt)} \psi(t)dt \geq \int_0^{\substack{N(Az,Sz,t)\diamond N(BBx_{2n-1},TBx_{2n-1},t)\diamond N(Sz,TBx_{2n-1},t)\diamond \\ N(Az,TBx_{2n-1},\propto t)\diamond N(Bz,Tz,(2-\propto)t)}} \psi(t)dt$$

Again taking limit as $n\rightarrow\infty$ and using (4), we have

$$\int_0^{N(Az,Tz,kt)} \psi(t)dt \leq \int_0^{\substack{N(Az,Sz,t)\diamond N(Tz,Tz,t)\diamond N(Sz,Tz,t)\diamond \\ N(Az,Tz,(1-q)t)\diamond N(Bz,Tz,(2-(1-q)t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{0\diamond 0\diamond N(Sz,Sz,t)\diamond \\ N(Az,Tz,t)\diamond N(Bz,Tz,t)\diamond N(Bz,Tz,qt)}} \psi(t)dt$$

$$\leq \int_0^{\substack{0\diamond 0\diamond 0 \\ N(Az,Tz,t)\diamond N(Bz,Tz,t)}} \psi(t)dt$$

$$\leq \int_0^{N(Az,Tz,t)\diamond 0} \psi(t)dt$$

$$\leq \int_0^{N(Az,Tz,t)} \psi(t)dt$$

Hence Az = Tz

From (ii), we get

$$\int_0^{M(Az,Bz,kt)} \psi(t)dt \geq \int_0^{\substack{M(Az,Sz,t)*M(Bz,Tz,t)*M(Sz,Tz,t)* \\ M(Az,Tz,\propto t)*M(Bz,Tz,(2-\propto t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{M(Tz,Tz,t)*M(Bz,Tz,t)*M(Tz,Tz,t)* \\ M(Tz,Tz,(1-q)t)*M(Bz,Tz,(2-(1-q)t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{M(Tz,Tz,t)*M(Bz,Tz,t)*M(Tz,Tz,t)* \\ M(Tz,Tz,t)*M(Bz,Tz,t)*M(Bz,Tz,qt)}} \psi(t)dt$$

$$\geq \int_0^{1*M(Bz,Tz,t)*1*1} \psi(t)dt$$

$$\geq \int_0^{M(Bz,Az,t)} \psi(t)dt$$

$$\geq \int_0^{M(Bz,Az,t)} \psi(t)dt$$

and

$$\int_0^{N(Az,Bz,kt)} \psi(t)dt \leq \int_0^{\substack{N(Az,Sz,t)\diamond N(Bz,Tz,t)\diamond N(Sz,Tz,t)\diamond \\ N(Az,Tz,\alpha t)\diamond N(Bz,Tz,(2-\alpha t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{N(Tz,Tz,t)\diamond N(Bz,Tz,t)\diamond N(Tz,Tz,t)\diamond \\ N(Tz,Tz,(1-q)t)\diamond N(Bz,Tz,(2-(1-q)t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{N(Tz,Tz,t)\diamond N(Bz,Tz,t)\diamond N(Tz,Tz,t)\diamond \\ N(Tz,Tz,t)\diamond N(Bz,Tz,t)\diamond N(Bz,Tz,qt)}} \psi(t)dt$$

$$\leq \int_0^{0\diamond N(Bz,Tz,t)\diamond 0\diamond 0} \psi(t)dt$$

$$\leq \int_0^{N(Bz,Az,t)} \psi(t)dt$$

$$\leq \int_0^{N(Az,Bz,t)} \psi(t)dt$$

Az = Bz

Therefore Az = Bz = Tz = Sz

Now, we show that Bz = z. From (ii), we get

$$\int_0^{M(Ax_{2n},Bz.kt)} \psi(t)dt \geq \int_0^{\substack{M(Ax_{2n},Sx_{2n},t)*M(Bz,Tz,t)*M(Sx_{2n},Tz,t)* \\ M(Ax_{2n},Tz,t)*M(Bx_{2n},Tx_{2n},t)*M(Bx_{2n},Tx_{2n},qt)}} \psi(t)dt$$

And, taking limit as $n\to\infty$, we have

$$\int_0^{M(z,Bz,kt)} \psi(t)dt \geq \int_0^{\substack{M(z,z,t)*M(Bz,Tz,t)*M(z,Tz,t)* \\ M(Tz,Tz,t)*M(z,z,t)*M(z,z,t)*M(z,z,t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{1*M(Bz,Tz,t)*M(z,Tz,t)* \\ 1*1*1}} \psi(t)dt$$

$$\geq \int_0^{M(Bz,Tz,t)*M(z,Tz,t)} \psi(t)dt$$

$$\geq \int_0^{M(Tz,Tz,t)*M(z,Bz,t)} \psi(t)dt$$

$$\geq \int_0^{M(z,Bz,t)} \psi(t)dt$$

And

$$\int_0^{N(z,Bz,kt)} \psi(t)dt \geq \int_0^{N(z,z,t)\diamond N(Bz,Tz,t)\diamond N(z,Tz,t)\diamond N(Tz,Tz,t)\diamond N(z,z,t)\diamond N(z,z,t)\diamond N(z,z,t)} \psi(t)dt$$

$$\geq \int_0^{0\diamond N(Bz,Tz,t)\diamond N(z,Tz,t)\diamond 0\diamond 0\diamond 0} \psi(t)dt$$

$$\geq \int_0^{N(Bz,Tz,t)\diamond N(z,Tz,t)} \psi(t)dt$$

$$\geq \int_0^{N(Tz,Tz,t)\diamond N(z,Bz,t)} \psi(t)dt$$

$$\geq \int_0^{N(z,Bz,t)} \psi(t)dt$$

And hence Bz = z.

Thus we get z = Az = Bz = Tz = Sz and so z is a common fixed point of A, B, S and T.

Uniqueness: In order to prove the uniqueness of fixed point, let u be another common fixed point of A, B, S and T. Then Au = Bu = Su = Tu, therefore, using (ii), we get

$$\int_0^{M(z,w,t)} \psi(t)dt = \int_0^{M(Az,Bw,kt)} \psi(t)dt \geq \int_0^{M(Az,Sz,t)*M(Bw,Tw,t)*M(Sz,Tw,t)*M(Az,Tw,\propto t)*M(Bz,Tz,(2-\propto)t)} \psi(t)dt$$

$$\int_0^{M(Az,Bw,kt)} \psi(t)dt \geq \int_0^{M(z,w,t)} \psi(t)dt$$

$$\int_0^{N(z,w,t)} \psi(t)dt = \int_0^{N(Az,Bw,kt)} \psi(t)dt\text{N} \leq \int_0^{N(Az,Sz,t)\diamond N(Bw,Tw,t)\diamond N(Sz,Tw,t)\diamond N(Az,Tw,\propto t)\diamond N(Bz,Tz,(2-\propto)t)} \psi(t)dt$$

$$\int_0^{N(Az,Bw,kt)} \psi(t)dt \leq \int_0^{N(z,w,t)} \psi(t)dt$$

From lemma 2.2, we get $z = w$

This completes the proof of theorem.

Corollary: 3.1

Let (X, M, N, $*$, $\diamond$) be a complete Intuitionistic fuzzy metric space and A, S be self mapping of X satisfying the following conditions:
(i) A(X) ⊆S(X)

(ii) $\int_0^{M(Ax,Ay,kt)} \psi(t)dt \geq \int_0^{M(Ax,Sx,t)*M(Ay,Sy,t)*M(Sx,Sy,t)*M(Ax,Sy,\alpha t)*M(Ax,Sx,(2-\alpha)t)} \psi(t)dt$

$\int_0^{N(Ax,By,kt)} \psi(t)dt \leq \int_0^{N(Ax,Sx,t)\diamond N(Ay,Sy,t)\diamond N(Sx,Sy,t)\diamond N(Ax,Sy,\alpha t)\diamond N(Ax,Sx,(2-\alpha)t)} \psi(t)dt$

Where α : [0, 1] → [0, 1] is continuous function such that $r(t) > t$ for some $0 < t < 1$ for all x, y $\in$ X, $k \in (0,1)$, $\alpha \in (0,2)$ and

(iii) S is continuous.

If (A, S) compatible of type of (K), then A, S have a unique common fixed point.

Corollary: 3.2

Let (X, M, N, $*$, $\diamond$) be a complete Intuitionistic fuzzy metric space and A, B, S and T be self mapping of X satisfying the following conditions:
(i) A(X) ⊆ T(X) and B(X) ⊆ S(X)

(ii) $\int_0^{M(Ax,By,kt)} \psi(t)dt \geq \int_0^{M(Ax,Sx,t)*M(Bx,Ty,t)*M(Sx,Ty,t)*M(Ax,Ty,t)} \psi(t)dt$

$\int_0^{N(Ax,By,kt)} \psi(t)dt \leq \int_0^{N(Ax,Sx,t)\diamond N(Bx,Ty,t)\diamond N(Sx,Ty,t)\diamond N(Ax,Ty,t)} \psi(t)dt$

Where α : [0, 1] → [0, 1] is continuous function such that $r(t) > t$ for some t > 1 for all x, y $\in$ X, $k \in (0,1)$ and

(iii) S and T are continuous.

If (A, S) and (B, T) compatible of type of (K), then A, B, S and T have a unique common fixed point.

Corollary: 3.3

Let (X, M, N, $*$, $\diamond$) be a complete Intuitionistic fuzzy metric space and A, B, S and T be self mapping of X satisfying conditions of our main theorem
If we putting $\propto = 1$ in (ii) condition of our main theorem, we have

(ii) $\int_0^{M(Ax,By,kt)} \psi(t)dt \geq \int_0^{\substack{M(Ax,Sx,t)*M(By,Ty,t)*M(Sx,Ty,t)* \\ M(Ax,Ty,t)*M(Bx,Tx,t)}} \psi(t)dt$

$\int_0^{N(Ax,By,kt)} \psi(t)dt \leq \int_0^{\substack{N(Ax,Sx,t)\diamond N(By,Ty,t)\diamond N(Sx,Ty,t)\diamond \\ N(Ax,Ty,t)\diamond N(Bx,Tx,t)}} \psi(t)dt$

Where $\propto$: [0, 1] → [0, 1] is continuous function such that $r(t) > t$ for some t > 1 for all x, y $\in$ X , $k \in (0,1)$ and

If (A, S) and (B, T) have a coincidence point .Moreover A, B, S and T have a unique common fixed point in X provided both the pairs (A, S) and (B, T) are compatible of type of (K).

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