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Reliability Characteristics of a Standby System with Priority for Preventive Maintenance and Server Failure

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ABSTRACT

Here, reliability characteristics of a two unit cold standby system are obtained by introducing the concepts of preventive maintenance, priority and server failure. The units are identical in nature and have two modes-normal and complete failures. There is a single server who attends the system immediately to tackle the faults whenever occur in the system. The preventive maintenance of the operating unit is conducted before its failure. The repair of the unit is done by the server at its complete failure. The server is subjected to failure while performing jobs. The failed server undergoes for treatment in order to resume the jobs. The random variables associated with failure time of the unit, preventive maintenance time and treatment time of the server are statistically independent with different probability distributions. Priority is given to the preventive maintenance over repair of the unit. The system model is analyzed by using semi-Markov process and regenerative point technique. The variation of some important reliability measures including mean time to system failure (MTSF), availability and profit function has been examined in steady state for arbitrary values of the parameters. The effect of the repair policy on profit of the system model is observed.

Keywords: Preventive Maintenance, Priority, server failure, Repair and Reliability Characteristics

I.INTRODUCTION

The breakdown of a service facility has been considered as a marked and unavoidable phenomenon in the queuing system. Until the failed facility is recovered, the waiting time for a customer in the system increases, thereby resulting a loss to the system. To offset this, an immediate treatment to the failed service facility is required. The systems with repairable server have been studied as queuing models by many authors such as Murari and Goyal (1984), Cao and Wu (1989) and Malik (2013). But, in most of these systems, it is assumed that repair facility neither fails nor deteriorates during jobs. In fact, this assumption seems to unrealistic when repair facility meets with an accident due to the reasons such as mishandling of the system, unskilled knowledge about the functioning of the systems, electric shocks and carelessness on the part of the server, etc. In such a situation, treatment may be given to the server in order to resume the job and also to reduce the down time of the system. Dhankar and Malik (2010) studied a single unit system with server failure during repair. Also, the existing literature on reliability modeling of repairable systems indicate that preventive maintenance is helpful in reducing the deterioration rate of the systems working in different environmental conditions. Malik (2013) discussed reliability model of a computer system with cold standby redundancy under preventive maintenance and repair.

In the light of above observations, this paper has been designed for determining reliability characteristics of a cold standby system with the ideas of preventive maintenance, server failure and priority in repair discipline.The

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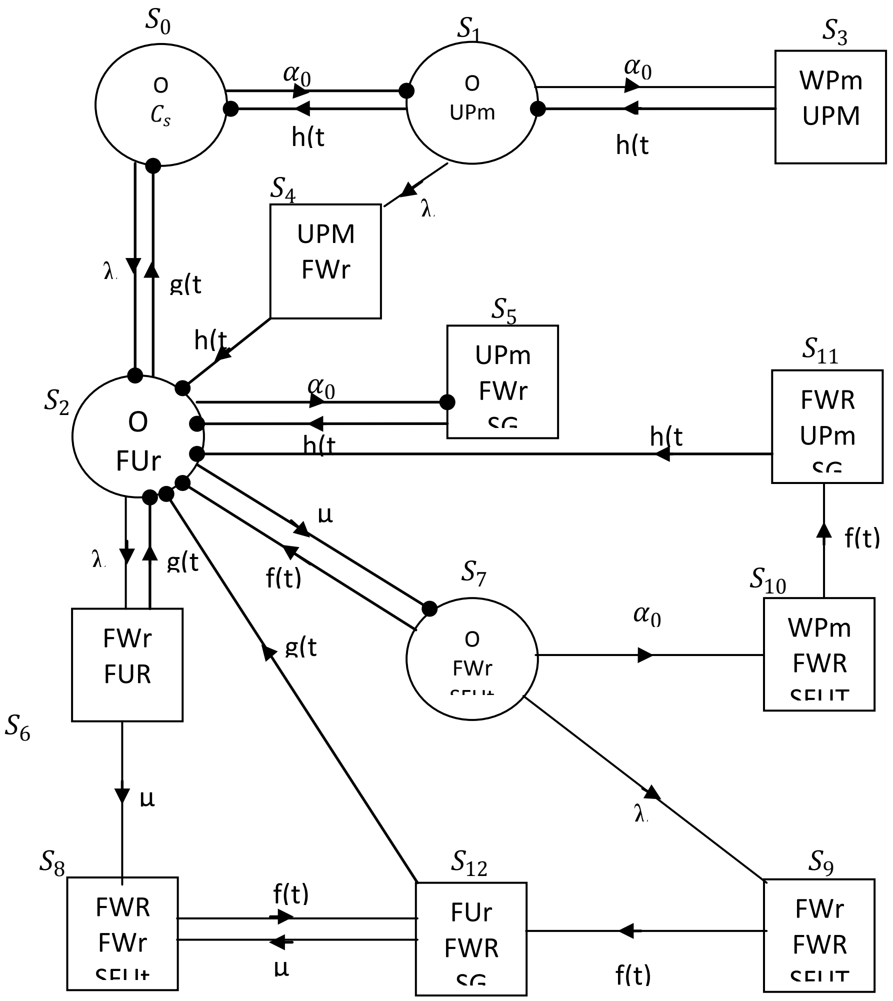
units are identical in nature and have two modes-normal and complete failures. There is a single server who attends the system immediately to tackle the faults whenever occur in the system. The preventive maintenance of the operating unit is conducted before its failure. The repair of the unit is done by the server at its complete failure. The server is subjected to failure while performing jobs. The failed server undergoes for treatment in order to resume the jobs. The random variables associated with failure time of the unit, preventive maintenance time and treatment time of the server are statistically independent with different probability distributions. Priority is given to the preventive maintenance over repair of the unit. The system model is analyzed by using semi-Markov process and regenerative point technique. The expressions for some important reliability measures including mean time to system failure (MTSF), availability, busy period of the server due to preventive maintenance and repair, expected number of preventive maintenances of the units and treatments give to the server and profit function are derivedin steady state and also for arbitrary values of the parameters. The effect of the repair policy on profit of the system model is observed.Graphs are drawn for a particular case to show the behavior of MTSF, availability and profit of the system model.

II.SYSTEM DESCRIPTION

A stochastic model for a system (cold standby)with priority to preventive maintenance over repair and server failure during repairhas been developed. The block diagram of the system model is shown in Fig.:1

Fig.1: State Transition



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The system mode (figure 1) has the following transition states:

- Regenerative States: $S_0, S_1, S_{2,} S_5 and\ S_7$
- Non-regenerative: $S_{3,}S_4,\ S_{6,} S_8, S_9, S_{10}, S_{11}\ and\ S_{12}$

The following are the possible transition states of the system:

S_0 = (O, Cs) S_1 = (O, UPm) S_2 = (O, FUr),

S_3= (WPm, UPM) S_4= (UPM, FWr) S_5= (UPm, FWr, SG)

S_6= (FWr, FUR) S_7= (O, FWr, SFUt) S_8=(FWR, FWr, SFUt)

S_9=(FWr, FWR, SFUT) S_{10}=(WPm, FWR, SFUT) S_{11}=(FWR, UPm, SG)

S_{12}=(FUr, FWR, SG)

III.TRANSITION PROBABILITIES (TP)

Simple probabilistic considerations yield the following expressions for the non-zero transition probabilities as:

$dQ_{01}(t) = \alpha_0 e^{-(\alpha_0+\lambda)t}dt$ $dQ_{02}(t) = \lambda e^{-(\alpha_0+\lambda)t}dt$

$dQ_{10}(t) = e^{-(\alpha_0+\lambda)t}h(t)dt$ $dQ_{13}(t) = \alpha_0 e^{-(\alpha_0+\lambda)t}\overline{H(t)}dt$

$dQ_{14}(t) = \lambda e^{-(\alpha_0+\lambda)t}\overline{H(t)}dt$ $dQ_{20}(t) = e^{-(\mu+\alpha_0+\lambda)t}g(t)dt$

$dQ_{25}(t) = \alpha_0 e^{-(\alpha_0+\lambda+\mu)t}\overline{G(t)}dt$ $dQ_{26}(t) = \lambda e^{-(\alpha_0+\lambda+\mu)t}\overline{G(t)}dt$

$dQ_{27}(t) = \mu e^{-(\alpha_0+\lambda+\mu)t}\overline{G(t)}dt$ $dQ_{31}(t) = h(t)dt$

$dQ_{42}(t) = h(t)dt$ $dQ_{52}(t) = h(t)dt$

$dQ_{62}(t) = e^{-\mu t}g(t)dt$ $dQ_{68}(t) = \mu e^{-\mu t}\overline{G(t)}dt$

$dQ_{62}(t) = e^{-\mu t}g(t)dt$ $dQ_{72}(t) = e^{-(\alpha_0+\lambda)t}f(t)dt$

$dQ_{7,9}(t) = \lambda e^{-(\alpha_0+\lambda)t}\overline{F(t)}dt$ $dQ_{7,10}(t) = \alpha_0 e^{-(\alpha_0+\lambda)t}\overline{F(t)}dt$

$dQ_{8,12}(t) = f(t)dt$ $dQ_{9,12}(t) = f(t)dt$

$dQ_{10,11}(t) = f(t)dt$ $dQ_{11,2}(t) = h(t)dt$

$dQ_{12,2}(t) = e^{-\mu t}g(t)dt$ $dQ_{12,8}(t) = \mu e^{-\mu t}\overline{G(t)}dt$

The transition probabilities can be obtained as:

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}\ (t)dt$$

We have

$p_{01} = \frac{\alpha_0}{\lambda+\alpha_0}$ $p_{02} = \frac{\lambda}{\lambda+\alpha_0}$ $p_{10} = h^*(\lambda+\alpha_0)$

$p_{13} = \frac{\alpha_0}{\lambda+\alpha_0}[1-h^*(\lambda+\alpha_0)]$ $p_{14} = \frac{\lambda}{\lambda+\alpha_0}[1-h^*(\lambda+\alpha_0)]$

$p_{20} = g^*(\mu+\lambda+\alpha_0)$ $p_{25} = \frac{\alpha_0}{\mu+\lambda+\alpha_0}[1-g^*(\mu+\lambda+\alpha_0)]$

$p_{26} = \frac{\lambda}{\mu+\lambda+\alpha_0}[1-g^*(\mu+\lambda+\alpha_0)]$ $p_{27} = \frac{\mu}{\mu+\lambda+\alpha_0}[1-g^*(\mu+\lambda+\alpha_0)]$

$p_{31} = h^*(0)$ $p_{42} = h^*(0)$

$p_{52} = h^*(0)$ $p_{62} = g^*(\mu)$

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$p_{68} = 1 - g^*(\mu)$ $p_{72} = f^*(\lambda + \alpha_0)$

$p_{7,9} = \frac{\lambda}{\lambda+\alpha_0}[1 - f^*(\lambda + \alpha_0)]$ $p_{7,10} = \frac{\alpha_0}{\lambda+\alpha_0}[1 - f^*(\lambda + \alpha_0)]$

$p_{8,12} = f^*(0)$ $p_{9,12} = f^*(0)$

$p_{10,11} = f^*(0)$ $p_{11,2} = h^*(0)$

$p_{12,2} = g^*(\mu)$ $p_{12,8} = 1 - g^*(\mu)$

For $h(t) = \gamma e^{-\gamma t}, g(t) = \beta e^{-\beta t} and f(t) = \alpha e^{-\alpha t}$ (1)

For a perfect distribution

$p_{01} + p_{02} = p_{10} + p_{13} + p_{14} = p_{20} + p_{25} + p_{26} + p_{27} = p_{31} = p_{42} = p_{52} = p_{62} + p_{68} = p_{72} + p_{79} + p_{7,10} =$
$p_{8,12} = p_{9,12} = p_{10,11} = p_{11,2} = p_{12,2} + p_{12,8} = 1$ (2)

1. **The Mean Sojourn Times (μ_i) is in the state S_i are**

$\mu_0 = \frac{1}{\alpha_0+\lambda}$ $\mu_1 = \frac{1}{\Upsilon+\alpha_0+\lambda}$ $\mu_2 = \frac{1}{\beta+\alpha_0+\alpha+\mu}$

$\mu_3 = \mu_4 = \mu_5 = \mu_{11} = \frac{1}{\Upsilon}$ $\mu_6 = \mu_{12} = \frac{1}{\beta+\mu}$ $\mu_7 = \frac{1}{\alpha+\alpha_0+\lambda}$

$\mu_8 = \mu_9 = \mu_{10} = \frac{1}{\alpha}$

$\mu_0 = m_{01} + m_{02}$ $\mu_1 = m_{10} + m_{13} + m_{14}$ $\mu_2 = m_{20} + m_{25} + m_{26} + m_{27}$

$\mu_3 = m_{31}$ $\mu_4 = m_{42}$ $\mu_5 = m_{52}$

$\mu_6 = m_{62} + m_{68}$ $\mu_7 = m_{72} + m_{7,9} + m_{7,10}$ $\mu_8 = m_{8,12}$

$\mu_9 = m_{9,12}$ $\mu_{10} = m_{10,11}$ $\mu_{11} = m_{11,2}$

$\mu_{12} = m_{12,2} + m_{12,8}$

$\mu_1' = m_{10} + m_{11.3} + m_{12.4}$

$\mu_2' = m_{20} + m_{22.6} + m_{22.6(8,12)^n} + m_{25} + m_{27}$

$\mu_7' = m_{72} + m_{72.10,11} + m_{72.9,12} + m_{72.9(12,8)^n}$ (3)

IV.RELIABILITY CHARACTERISTICS

The following reliability characteristics have been evaluated for the system model:

4.1 Mean Time To System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1 + Q_{02}(t)Ⓢ\phi_2$

$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0 + Q_{13}(t) + Q_{14}(t)$

$\phi_2(t) = Q_{20}(t)Ⓢ\phi_0 + Q_{27}(t)Ⓢ\phi_7 Q_{13}(t) + Q_{25}(t) + Q_{27}(t)$

$\phi_7(t) = Q_{72}(t)Ⓢ\phi_2 + Q_{7,10}(t) + Q_{7,9}(t)$ (4)

Taking LST of above relation (4) and solving for $\widetilde{\phi_0}(s)$, we have

$$R^*(s) = \frac{1-\widetilde{\phi_0}(s)}{s} \quad (5)$$

And

$$MTSF = \lim_{s\to 0} \frac{1-\widetilde{\phi_0}(s)}{s} = \frac{N_1}{D_1} \quad (6)$$

Where, $N_1 = (1-p_{27}p_{72})(\mu_0 + p_{01}\mu_1) + p_{02}(\mu_2 + p_{27}\mu_7)$

$D_1 = (1-p_{01}p_{10})(1-p_{27}p_{72}) - p_{02}p_{20}$

4.2 Long Run Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for $A_i(t)$ are given as under:

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{11.3}(t)©A_1(t) + q_{12.4}(t)©A_2(t)$

$$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + \left(q_{22.6}(t) + q_{22.6(8,12)^n}(t)\right)©A_2(t) + q_{25}(t)©A_5(t) + q_{27}(t)©A_7(t)$$

$A_5(t) = q_{52}(t)©A_2(t)$

$A_7(t) = M_7(t) + (q_{72}(t) + q_{72.10,11}(t) + q_{72.9,12}(t)+q_{72.9(12,8)^n}(t))©A_2(t) \quad (7)$

Where,

$M_0(t) = e^{-(\lambda+\alpha_0)t}$ $\qquad M_1(t) = e^{-(\lambda+\alpha_0)t}\overline{H(t)}$

$M_2(t) = e^{-(\mu+\lambda+\alpha_0)t}\overline{G(t)}$ $\qquad M_7(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)}$

Taking LT of above relations (7) and solving for $A_0^*(s)$.

The steady state availability is given by

$A_0(\infty) = \lim_{s\to 0} s\, A_0^*(s) = \frac{N_2}{D_2}$

Where, $N_2 = p_{20}(1-p_{13})\mu_0 + p_{01}p_{20}\mu_1 + (p_{10}p_{02} + p_{14})(\mu_2 + \mu_7 p_{27})$

$D_2 = p_{20}(1-p_{13})\mu_0 + p_{20}p_{01}\mu_1' + (p_{10}p_{02} + p_{14})(\mu_2' + p_{25}\mu_5 + p_{27}\mu_7')$

4.3 Busy Period of the Server Due to Repair in the Long Run

Let $B_i^R(t)$be the probability that the server is busy in repairing the unit due to long run at an instant 't' given that the system entered state S_i at t = 0. The recursive relations $B_i^R(t)$for are as follows:

$B_0^R(t) = q_{01}(t)©B_1^R(t) + q_{02}(t)©B_2^R(t)$

$B_1^R(t) = q_{10}(t)©B_0^R(t) + q_{11.3}(t)©B_1^R(t) + q_{12.4}(t)©B_2^R(t)$

$B_2^R(t) = W_2^R(t) + q_{20}(t)©B_0^R(t) + \left(q_{22.6}(t) + q_{22.6(8,12)^n}(t)\right)©B_2^R(t) + q_{25}(t)©B_5^R(t) + q_{27}(t)©B_7^R(t)$

$B_5^R(t) = q_{52}(t)©B_2^R(t)$

$B_7^R(t) = \left(q_{72}(t) + q_{72.10,11}(t) + q_{72.9,12}(t) + q_{72.9,11(12,8)^n}(t)\right)©B_2^R(t) \quad (8)$

where $W_i^R(t)$ be the probability that the server is busy in state S_i due to long run at an at time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

Where,$W_2^R(t) = e^{-(\alpha_0+\mu+\lambda)t}\overline{G(t)} + (\lambda e^{-\lambda t}©\mu e^{-\mu t}©f(t)©1)\overline{G(t)}$

Let us take LT of relations (8) and solving for$B_0^{R*}(s)$.

We get, $B_0^R(\infty) = \lim_{s\to 0} sB_0^{R*}(s) = \frac{N_3}{D_2}$

$N_3 = W_2^*(0)(p_{14} + p_{02}p_{10})andD_2$ is already specified.

4.4 Busy Period of the Server due to Preventive Maintenance (PM) in the Long Run

Let $B_i^P(t)$be the probability that the server is busy in repairing the unit due to preventive maintenance in the long run at an instant 't' given that the system entered state i at t = 0. The recursive relations $B_i^P(t)$for are as follows:

$B_0^P(t) = q_{01}(t)©B_1^P(t) + q_{02}(t)©B_2^P(t)$

$B_1^P(t) = W_1^P(t)+q_{10}(t)©B_0^P(t) + q_{11.3}(t)©B_1^P(t) + q_{12.4}(t)©B_2^P(t)$

$B_2^P(t) = q_{20}(t)©B_0^P(t) + \left(q_{22.6}(t) + q_{22.6(8,12)^n}(t)\right)©B_2^P(t) + q_{25}(t)©B_5^P(t) + q_{27}(t)©B_7^P(t)$

$B_5^P(t) = \left(q_{41}(t) + q_{41(8,12)^n}(t)\right)©B_1^P(t)$

$B_7^P(t) = \left(q_{72}(t) + q_{72.10,11}(t) + q_{72.9,12}(t) + q_{72.9(12,8)^n}(t)\right)©B_2^P(t)$(9)

where$W_i^P(t)$ be the probability that the server is busy in state S_i due to preventive maintenance in long runat time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

Where, $W_1^P(t) = e^{-(\alpha_0+\lambda)t}\overline{H(t)} + (\alpha_0 e^{-\alpha_0 t}©1)\overline{H(t)} + (\lambda e^{-\lambda t}©1)\,\overline{H(t)}$ and $W_5^P(t) = \overline{H(t)}$

Let us take LTofrelations (9) and solving for$B_0^{P*}(s)$.

We get, $B_0^P(\infty) = \lim_{s\to 0} sB_0^{P*}(s) = \frac{N_4}{D_2}$

$N_4 = W_1^{P*}(0)p_{01}p_{20} + W_5^{P*}(0)p_{01}p_{14}p_{25}andD_2$ is already specified.

4.5 Expected Number of Repairs (ENR) of the unit in the Long Run

Let $R_i(t)$ be the expected number of repair of the unit in the long run by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i(t)$ are given as:

$R_0(t) = Q_{01}(t)Ⓢ R_1(t) + Q_{02}(t)Ⓢ R_2(t)$

$R_1(t) = Q_{10}(t)Ⓢ R_0(t) + Q_{11.3}(t)Ⓢ R_1(t) + Q_{12.4}(t)©R_2(t)$

$R_2(t) = Q_{20}(t)Ⓢ(1 + R_0(t)) + \left(Q_{22.6}(t) + Q_{22.6(8,12)^n}(t)\right)Ⓢ\left(1 + R_1(t)\right) + Q_{25}(t)Ⓢ R_5(t)$
$\quad + Q_{27}(t)Ⓢ R_7(t)$

$R_5(t) = Q_{52}(t)Ⓢ R_2(t)$

$R_7(t) = \left(Q_{72}(t) + Q_{72.10,11}(t)\right)Ⓢ R_2(t) + \left(Q_{72.9,12}(t) + Q_{72.9(12,8)^n}(t)\right)Ⓢ\left(1 + R_2(t)\right)$ (10)

Taking LST of relations (10). And, solving for$\widetilde{PM}_i(s)$.The expected numbers of preventive maintenance of the unit in the long run are given by

Let us take LSTof relations (10) and solving for$\widetilde{R_0}(s)$

We get, $R_0 = \lim_{s\to 0} s\widetilde{R_0}(s) = \frac{N_5}{D_2}$

Where, $N_5 = (p_{14} + p_{10}p_{02})(p_{20} + p_{26} + p_{79}p_{27})$ and D_2 is already specified.

4.6 Expected Number of Preventive Maintenance (PM) of the unit in the Long Run

Let $PM_i(t)$ be the expected number of preventive maintenance of the unit in the long run by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $PM_i(t)$ are given as

$PM_0(t) = Q_{01}(t)ⓈPM_1(t) + Q_{02}(t)ⓈPM_2(t)$

$PM_1(t) = Q_{10}(t)Ⓢ(1 + PM_0(t)) + Q_{11.3}(t)Ⓢ(1 + PM_1(t)) + Q_{12.4}(t)Ⓢ(1 + PM_2(t))$

$PM_2(t) = Q_{20}(t)ⓈPM_6(t)) + \left(Q_{22.6}(t) + Q_{22.6(8,12)^n}(t)\right)ⓈPM_2(t) + Q_{25}(t)ⓈPM_5(t) + Q_{27}(t)ⓈPM_7(t)$

$PM_5(t) = Q_{52}(t)Ⓢ(1 + PM_2(t))$

$PM_7(t) = Q_{72}(t)ⓈPM_2(t) + \left(Q_{72.9,12}(t) + Q_{72.9(12,8)^n}(t)\right)ⓈPM_2(t) + Q_{72.10,11}(t)Ⓢ(1 + PM_2(t))$

(11)

Taking LST of relations (11). And, solving for$\widetilde{PM}_i(s)$.The expected numbers of preventive maintenance of the unit in the long run are given by

We get, $PM_0 = \lim_{s\to 0} s\widetilde{PM}_i(s) = \frac{N_6}{D_2}$

Where, $N_6 = p_{01}p_{20} + (p_{14} + p_{02}p_{10})(p_{25} + p_{7,10}p_{27})$ and D_2 is already specified.

4.7 Expected Number of Treatments (ENT) Given to the Server in the Long Run

Let $T_i(t)$ be the expected number of treatment given to the server in the long run in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $T_i(t)$ are given as

$T_0(t) = Q_{01}(t)ⓈT_1(t) + Q_{02}(t)ⓈT_2(t)$

$T_1(t) = Q_{10}(t)ⓈT_0(t) + Q_{11.3}(t)ⓈT_1(t) + Q_{12.4}(t)ⓈT_2(t)$

$T_2(t) = Q_{20}(t)ⓈT_0(t) + Q_{22.6}(t)ⓈT_2(t) + Q_{22.6(8,12)^n}(t)Ⓢ(1 + T_2(t)) + Q_{25}(t)ⓈT_5(t) + Q_{27}(t)ⓈT_7(t)$

$T_5(t) = Q_{52}(t)ⓈT_2(t)$

$T_7(t) = (Q_{72}(t) + Q_{72.10,11}(t) + Q_{72.9,12}(t) + Q_{72.9(12,8)^n}(t))Ⓢ(1 + T_2(t))$ (12)

Taking LST of relations (12). And, solving for $\widetilde{T}_0(s)$The expected numbers of preventive maintenance of the unit in the long run are given by

Let us take LST of relations (12) and solving for$\widetilde{T}_0(s)$.

We get, $T_0 = \lim_{s\to 0} s\widetilde{T}_0(s) = \frac{N_7}{D_2}$

Where, $N_7 = (p_{14} + p_{02}p_{10})(p_{27} + p_{26}p_{68})$ and D_2 is already specified.

2. Profit Analysis

$P = K_0A_0 - K_1B_0^R - K_4B_0^P - K_2R_0 - K_5P_0 - K_3T_0$

The profit of the system model can be obtained as:

Where

P = Profit of the system model

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to repair

K_2 = Cost per unit time repair

K_3 = Cost per unit time for treatment of the server

K_4 =Cost per unit for which server is busy due to preventive maintenance

K_5 = Cost per unit time repair of the unit

3. Particular Cases

Let us take $g(t) = \beta e^{-\beta t}$ $f(t) = \alpha e^{-\alpha t}$ $h(t) = Ye^{-Yt}$

$p_{01} = \frac{\alpha_0}{\lambda+\alpha_0}$ $p_{02} = \frac{\lambda}{\lambda+\alpha_0}$ $p_{10} = \frac{Y}{Y+\lambda+\alpha_0}$

$p_{13} = \frac{\alpha_0}{Y+\lambda+\alpha_0}$ $p_{14} = \frac{\lambda}{Y+\lambda+\alpha_0}$ $p_{20} = \frac{\beta}{\beta+\mu+\lambda+\alpha_0}$

$p_{25} = \frac{\alpha_0}{\beta+\mu+\lambda+\alpha_0}$ $p_{26} = \frac{\lambda}{\beta+\mu+\lambda+\alpha_0}$ $p_{27} = \frac{\mu}{\beta+\mu+\lambda+\alpha_0}$

$p_{31} = p_{42} = p_{52} = p_{8,12} = p_{9,12} = p_{10,11} = p_{11,2} = 1$

$p_{62} = \frac{\beta}{\beta+\mu}$ $p_{68} = \frac{\mu}{\beta+\mu}$ $p_{72} = \frac{\alpha}{\alpha+\lambda+\alpha_0}$

$p_{7,9} = \frac{\lambda}{\alpha+\lambda+\alpha_0}$ $p_{7,10} = \frac{\alpha_0}{\alpha+\lambda+\alpha_0}$ $p_{12,2} = \frac{\beta}{\beta+\mu}$

$p_{12,8} = \frac{\mu}{\beta+\mu}$ $\mu_0 = \frac{1}{\lambda+\alpha_0}$ $\mu_1 = \frac{1}{Y+\lambda+\alpha_0}$ $\mu_2 = \frac{1}{\beta+\mu+\lambda+\alpha_0}$ $\mu_3 = \mu_4 = \mu_5 = \mu_{11} = \frac{1}{Y}$

$\mu_6 = \mu_{12} = \frac{1}{\beta+\mu}$

$\mu_7 = \frac{1}{\alpha+\lambda+\alpha_0}$ $\mu_8 = \mu_9 = \mu_{10} = \frac{1}{\alpha}$

$\mu_1' = \frac{1}{Y}$ $\mu_2' = \frac{\alpha\beta+\lambda(\alpha+\mu)}{\alpha\beta(\beta+\mu+\lambda+\alpha_0)}$ $\mu_7' = \frac{\beta\{\alpha Y+\alpha_0(\alpha+Y)\}+Y\lambda(\beta+\alpha+\mu)}{\alpha\beta Y(\alpha+\lambda+\alpha_0)}$

$MTSF = \frac{N_1}{D_1}$, $Availability(A_0) = \frac{N_2}{D_2}$

$B_0^R = \frac{N_3}{D_2}$ $B_0^P = \frac{N_4}{D_2}$ $R_0 = \frac{N_5}{D_2}$

$P_0 = \frac{N_6}{D_2}$ $T_0 = \frac{N_7}{D_2}$

Where,

$$N_1 = (\gamma+\lambda+2\alpha_0)[(\beta+\lambda+\mu+\alpha_0)(\alpha+\lambda+\alpha_0)-\mu\alpha]+\lambda(\gamma+\lambda+\alpha_0)(\alpha+\lambda+\mu+\alpha_0)$$

$$D_1 = [(\lambda+\alpha_0)(\gamma+\lambda+\alpha_0)-\gamma\alpha_0][(\beta+\lambda+\mu+\alpha_0)(\alpha+\lambda+\alpha_0)-\mu\alpha]-\lambda\beta(\gamma+\lambda+\alpha_0)(\alpha+\lambda+\alpha_0)$$

$$N_2 = \frac{\lambda\mu+(\beta+\lambda)(\alpha+\lambda+\alpha_0)}{(\lambda+\alpha_0)(\alpha+\lambda+\alpha_0)(\beta+\mu+\lambda+\alpha_0)}$$

$$D_2 = \frac{\alpha\beta^2 Y(\alpha+\lambda+\alpha_0)(Y+\lambda)+\alpha_0\beta^2\alpha(\alpha+\lambda+\alpha_0)(\gamma+\lambda+\alpha_0)+\lambda(\gamma+\lambda+\alpha_0)[Y(\alpha+\lambda+\alpha_0)\{\beta\alpha+\lambda(\alpha+\mu)\}+\alpha_0\beta\alpha(\alpha+\lambda+\alpha_0)+\mu\{\alpha\beta Y+\alpha_0\beta(Y+\alpha)+\lambda Y(\alpha+\beta+\mu)\}]}{\alpha\beta Y(\lambda+\alpha_0)(\alpha+\lambda+\alpha_0)(\gamma+\lambda+\alpha_0)(\beta+\mu+\lambda+\alpha_0)}$$

$$N_3 = \frac{\lambda[\beta(\mu+\alpha_0)+\lambda(\beta+\mu+\lambda+\alpha_0)]}{\beta(\lambda+\alpha_0)(\mu+\lambda+\alpha_0)(\beta+\mu+\lambda+\alpha_0)}$$

$$N_4 = \frac{\alpha_0[(\gamma+\alpha_0+\lambda)Y+\alpha_0\lambda]}{Y(\lambda+\alpha_0)(\gamma+\alpha_0+\lambda)(\beta+\mu+\lambda+\alpha_0)}$$

$$N_5 = \frac{\lambda[\lambda\mu+(\beta+\lambda)(\alpha+\lambda+\alpha_0)]}{(\lambda+\alpha_0)(\alpha+\alpha_0+\lambda)(\beta+\mu+\lambda+\alpha_0)}$$

$$N_6 = \frac{\alpha_0[\lambda\mu + (\beta+\lambda)(\alpha+\lambda+\alpha_0)]}{(\lambda+\alpha_0)(\alpha+\alpha_0+\lambda)(\beta+\mu+\lambda+\alpha_0)}$$

$$N_7 = \frac{\lambda\,\mu(\beta+\mu+\lambda)}{(\lambda+\alpha_0)(\beta+\mu)(\beta+\mu+\lambda+\alpha_0)}$$

Numerical and Graphical Representation of Reliability Measures

Table 1: MTSF Vs Failure Rate

λ	β=3.1,α=2.1, μ=0.2,Ƴ=2.1, α0=2.2	α=3.1	β=4.1	μ=0.4	α0=2.6	Ƴ=2.7
0.1	1.2725	1.2727	1.2773	1.27179	1.0326	1.3820
0.2	1.2086	1.2088	1.2172	1.2073	0.9890	1.3058
0.3	1.1503	1.1506	1.1621	1.1486	0.9487	1.2369
0.4	1.0971	1.0974	1.1112	1.0950	0.9114	1.1746
0.5	1.0482	1.0485	1.0644	1.0459	0.8766	1.1178
0.6	1.0032	1.0036	1.0209	1.0008	0.8444	1.0659
0.7	0.9616	0.9621	0.9807	0.9592	0.8142	1.0184
0.8	0.9232	0.9236	0.9432	0.9206	0.7861	0.9747
0.9	0.8875	0.8879	0.9083	0.8849	0.7596	0.9344

Table 2: Availability Vs Failure Rate

Λ	β=3.1,μ=0.2,Ƴ=2.1, α=2.1,α0=2.2	α=3.1	β=4.1	μ=0.4	α0=2.6	Ƴ=2.7
0.1	0.638309857	0.638662	0.640246	0.637339	0.582459	0.718455
0.2	0.626125291	0.626811	0.629969	0.624225	0.571884	0.705116
0.3	0.614441143	0.615443	0.62016	0.611655	0.561737	0.692131
0.4	0.603214669	0.604515	0.610769	0.599587	0.551982	0.679492
0.5	0.592409414	0.593991	0.601758	0.587983	0.542589	0.667192
0.6	0.581994007	0.583841	0.59309	0.576812	0.533531	0.655225
0.7	0.571941228	0.574037	0.584737	0.566045	0.524785	0.643582
0.8	0.56222726	0.564557	0.576672	0.555658	0.516329	0.632255
0.9	0.552831108	0.555381	0.568873	0.545628	0.508146	0.621236

Table 3: Profit Vs Failure Rate

λ	β=3.1,μ=0.2,Ƴ=2.1, α=2.1,α0=2.2	α=3.1	β=4.1	μ=0.4	α0=2.6	Ƴ=2.7
0.1	8986.764272	8991.659	9072.41	8971.397	8123.667	10116.61
0.2	8692.01388	8701.391	8816.653	8662.471	7863.546	9791.326

0.3	8409.086428	8422.557	8571.125	8366.511	7613.585	9476.132
0.4	8137.066958	8154.262	8334.898	8082.55	7373.008	9170.712
0.5	7875.166034	7895.739	8107.179	7809.744	7141.143	8874.751
0.6	7622.696191	7646.32	7887.285	7547.352	6917.4	8587.939
0.7	7379.053534	7405.422	7674.624	7294.715	6701.26	8309.971
0.8	7143.703237	7172.53	7468.679	7051.244	6492.264	8040.549
0.9	6916.168028	6947.184	7268.998	6816.41	6289.998	7779.379

V.CONCLUSION

The numerical results considering a particular case are obtained for some reliability characteristics of the system model. It is observed that the MTSF (shown in table1) keeps on decline with the increase of the failure rate of the unit and server and the rate at which unit goes on Preventive Maintenance while it increases with the increase of treatment rate of the server (α), repair rate (β) and preventive maintenance rate of the unit (γ). The results given in tables 2 & 3 indicate that availability and profit go on decreasing when we increase of failure rate of the unit and server as well as the rate by which unit under goes for preventive maintenance. However, their values keep on increasing with the increase of repair rate of the unit, treatment rate of the server and preventive maintenance rate of the unit. The study also reveals that the provision of priority to preventive maintenance over repair is helpful in making the system more profitable and practicable to use.

REFERENCES